L3 Mention Informatique
Parcours Informatique et MIAGE

Génie Logiciel Avancé - Advanced Software Engineering

Deductive Verification I

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Recall: Validation and Verification

- Validation :
  - Does the system meet the clients requirements ?
  - Will the performance be sufficient ?
  - Will the usability be sufficient ?

Do we build the right system ?

- Verification: Does the system meet the specification ?

Do we build the system right ?

Is it « correct » ?

Recall: What are the limits of tests

- Assumptions on „Testability"

  (system under test must behave deterministically,
  or have controlled non-determinism, must be initializable)

- Assumptions like Test-Hypothesis

  (Uniform / Regular behaviour is sometimes
  a „realistic" assumption, but not always)

- Limits in perfection:

  We know only up to a given "certainty" that the
  program meets the specification ...

The role of formal proof

- formal proofs are another technique for program verification
  - based on a model of the underlying programming language,
    the conformance of a concrete program to its specification
    can be established

    FOR ALL INPUT DATA AND ALL INITIAL STATES !!!

- formal proofs as verification technique can:
  - verify that a more concrete design-model "fits"
    to a more abstract design model
    (construction by formal refinement)

  - verify that a program "fits" to a concrete design model.

Who is using formal proofs in industry?

- Hardware Suppliers:
  - INTEL: Proof of Floating Point Computation compliance to IEEE754
  - INTEL: Correctness of Cash-Memory-Coherence Protocols
  - AMD: Correctness of Floating-Point-Units againt Design-Spec
  - GemPlus: Verification of Smart-Card-Applications in Security
- Software Suppliers:
  - MicroSoft: Many Drivers running in „Kernel Mode" were verified
  - MicroSoft: Verification of the Hyper-V OS (60000 Lines of Concurrent, Low-Level C Code ...)
  - . . .

Who is using formal proofs in industry?

- For the highest certification levels along the lines of the Common Criteria, formal proofs are
  - recommended (EAL6)
  - mandatory (EAL7)

  There had been now several industrial cases of EAL7 certifications ...

- For lower levels of certifications, still, formal specifications were required.

- Recently, Microsoft has agreed in a Monopoly-Lawsuit against the European Commission to provide a formal Spec of the Windows-Server-Protocols
  - the tools validating them use internally automated proofs

Pre-Rerquisites of Formal Proof Techniques

- A Formal Specification (MOAL, HOL, but also Z, VDM, CSP, B, ...)
  - know-how over the application domain
  - informal and formal requirements of the system
- Either a formal model of the programming language
  or a trusted code-generator from concrete design specs
- Tool Chains to generate, simplify, and solve large formulas
  (decision procedures)
- Proof Tools and Proof Checker: proofs can also be false ...

Nous, on le fera à la main ...

How to do Verification ?

In the sequel, we concentrate on

Deductive Verification

(Proof Techniques)

Standard example

The specification in UML/MOAL (Classes in USE Notation):

```
class Triangles inherits_from Shapes
   attributes
      a : Integer
      b : Integer
      c : Integer

   operations
      mk(Integer,Integer,Integer):Triangle
      is_Triangle(): triangle
   end
```

Standard example : Triangle

The specification in UML/OCL (Classes in USE Notation):

```
context Triangles:
inv def      : a.oclIsValid() and b.oclIsValid()...
inv pos      : 0<a and 0<b and 0<c
inv triangle : a+b>c  and  b+c>a  and  c+a>b

context Triangle::isTriangle()
post equi : a=b and b=c implies result=equilateral
post iso  : ((a<>b or b<>c) and
             (a=b or b=c or a=c))implies result=isosceles
post default: (a<>b or b<>c) and
              (a<>b and b<>c and a<>c)
              implies result=arbitrary
```

Standard example: Triangle

```
procedure triangle(j,k,l : positive) is
 eg: natural := 0;
begin
if  j + k <= l or k + l  <= j or l + j <= k then
    put("impossible");
else if  j = k  then   eg := eg + 1;  end if;
     if  j = l  then   eg := eg + 1;  end if;
     if  l = k  then   eg := eg + 1;  end if;
      if  eg = 0  then  put("quelconque");
     elsif   eg = 1  then put("isocele");
     else    put("equilateral");
     end if;
end if;
end triangle;
```

Program Example : Exponentiation

Program_1 :

```
(* pre : N≥0 *)
S:=1; P:=N;
while P >= 1 loop S:= S*X; P:= P-1; end loop;
(* post: S = X^N *)
```

Program_2 :

```
(* pre : N≥0 *)
S:=1; P:= N;
while P >= 1 loop
    if P mod 2 <> 0 then P := P-1; S := S*X; end if;
    S:= S*S; P := P div 2;
end loop;
(* post: S = X^N *)
```

These programs have the following characteristics:

- one is more efficient, but more complex
- But both have the same specification !

How to do Verification ?

How to PROVE that programs

meet the specification ?

Foundations: Proof Systems

- An Inference System (or *Logical Calculus*) allows to infer formulas from a set of *elementary facts* (axioms) and inferred facts by rules:

$$\frac{A_1 \quad \ldots \quad A_n}{A_{n+1}}$$

- "from the *assumptions* A_1 to A_n, you can infer the *conclusion* A_{n+1}."

  A rule with n=0 is an *elementary fact*. Variables occurring in the formulas A_n can be arbitrarily substituted.

- Assumptions and conclusions are terms in a logic containing variables

Foundations: Proof Systems

- An Inference System for the equality operator (or "Equational Logic") looks like this:

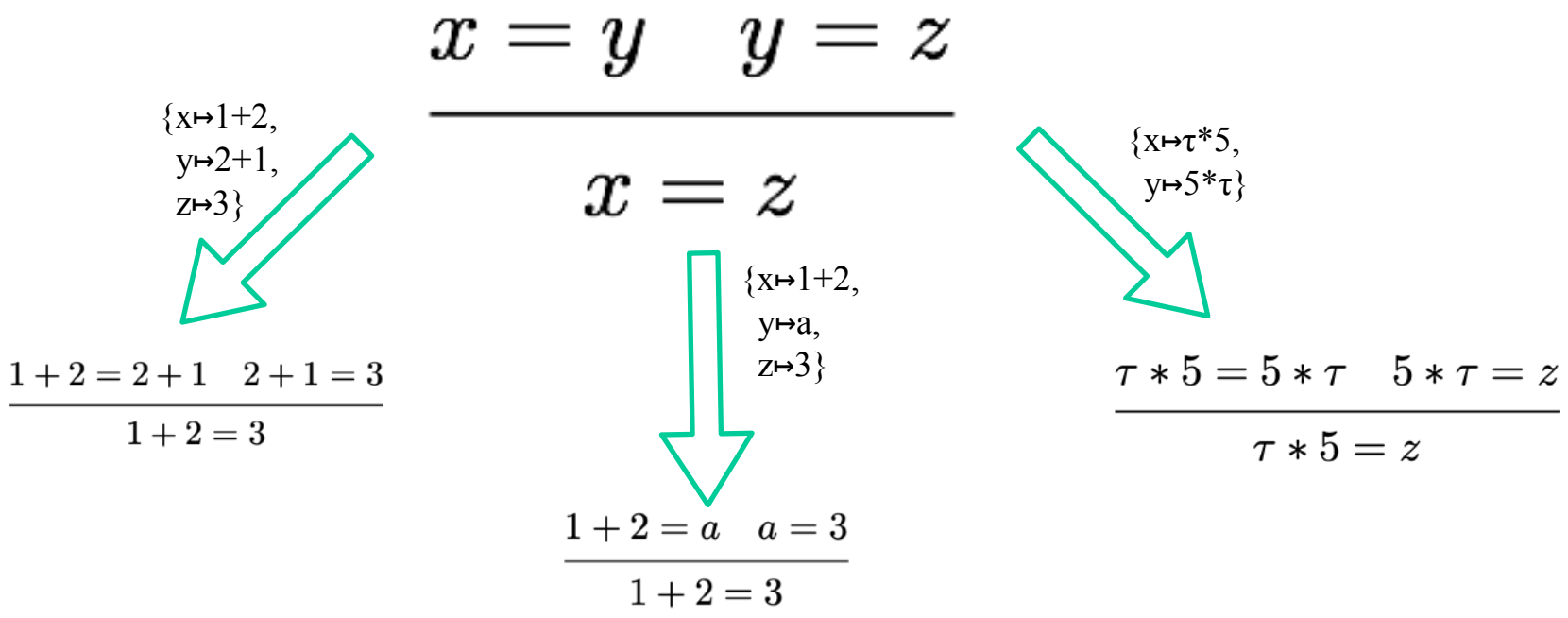
$$\frac{}{x = x} \qquad \frac{x = y}{y = x} \qquad \frac{x = y \quad y = z}{x = z}$$

$$\frac{x = y \quad P(x)}{P(y)}$$

- where the first rule "reflexivity" is an elementary fact.

Foundations: Proof Systems

The variables in an inference rule can be replaced by a substitution. The substituted inference rule is called an instance (of this rule).



Foundations: Proof Systems

- A *Formal Proof* (or : *Derivation*)
  is a tree with rule instances as nodes

$$\dfrac{\dfrac{f(a,b)=a}{a=f(a,b)} \quad \dfrac{f(a,b)=a \quad f(f(a,b),b)=c}{f(a,b)=c}}{a=c} \quad \dfrac{}{g(a)=g(a)}$$
$$\dfrac{}{g(a)=g(c)}$$

- The non-elementary facts at the leaves are the *global assumptions* (here $f(a,b) = a$ and $f(f(a,b),b) = c$).

Foundations: Proof Systems

- As a short-cut, we also write for a derivation:

$$\{A_1, \ldots, A_n\} \vdash A_{n+1}$$

- ... or generally speaking: from global assumptions A to a theorem (in theory E) φ:

$$\Gamma \vdash_E \phi$$

- This is what theorems are: derivable facts from assumptions in a certain logical system ...

A Proof System for Propositional Logic

- PL + E + Arithmetics (A) in so-called natural deduction:

$$\frac{\neg\neg A}{A} \qquad \frac{A}{\neg\neg A}$$

$$\frac{A \quad B}{A \wedge B} \qquad \frac{A \wedge B \qquad \begin{matrix}[A, B] \\ \vdots \\ Q\end{matrix}}{Q} \qquad \frac{False}{A}$$

$$\frac{A}{A \vee B} \qquad \frac{B}{A \vee B} \qquad \frac{\begin{matrix}[A] \\ \vdots \\ \neg A\end{matrix}}{B} \qquad \frac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix}}{A \to B} \qquad \frac{P \to Q \quad P}{Q}$$

Hoare – Logic: A Proof System for Programs

- Now, can we build a

Logic for Programs ???

Hoare – Logic: A Proof System for Programs

- Now, can we build a

  Logic for Programs ???

  Well, yes !

  There are actually lots of possibilities ...

- We consider the Hoare-Logic (Sir Anthony Hoare ...), technically an inference system PL + E + A + Hoare

Hoare – Logic: A Proof System for Programs

- Basis: The mini-language „IMP“,
  (following Glenn Wynskell's Book)

- We have the following commands (*cmd*)
  - the empty command SKIP
  - the assignment x:== E (x ∈ V)
  - the sequential compos. c_1 ; c_2
  - the conditional IF cond THEN c_1 ELSE c_2
  - the loop WHILE cond DO c

  where c, c_1, c_2, are cmd's, V variables,

  E an arithmetic expression, and cond a boolean expression.

Hoare – Logic: A Proof System for Programs

- Core Concept: A Hoare Triple consisting …
  - of a pre-condition P
  - a post-condition Q
  - and a piece of program cmd
  - the triple (P,cmd,Q) is written:

$$\vdash \{P\}\ cmd\ \{Q\}$$

  - *P* and *Q* are formulas over the variables *V*, so they can be seen as set of possible states.

Hoare Logic vs. Symbolic Execution

- Hoare Logic is also based notion of a *symbolic state.*

$$state_{sym} = V \rightarrow Set(D)$$

Hoare Logic vs. Symbolic Execution

- Intuitively:

$$\vdash \{P\}\ cmd\ \{Q\}\ .$$

means:

If a program cmd starts in a state admitted by P if it terminates, that the program must reach a state that satisfies Q.

Hoare – Logic: A Proof System for Programs

- PL + E + A + Hoare (simplified binding) at a glance:

$$\frac{}{\vdash \{P\}\ \text{SKIP}\ \{P\}} \qquad \frac{}{\vdash \{P[x \mapsto E]\}\ \text{x :== E}\{P\}}$$

$$\frac{\vdash \{P \wedge cond\}\ c\ \{Q\} \quad \vdash \{P \wedge \neg cond\}\ d\ \{Q\}}{\vdash \{P\}\ \text{IF } cond \text{ THEN } c \text{ ELSE } d\{Q\}}$$

$$\frac{\vdash \{P\}\ c\ \{Q\} \quad \vdash \{Q\}\ d\ \{R\}}{\vdash \{P\}\ c;\ d\ \{R\}} \qquad \frac{\vdash \{P \wedge cond\}\ c\ \{P\}}{\vdash \{P\}\ \text{WHILE } cond \text{ DO } c\ \{P \wedge \neg cond\}}$$

$$\frac{P \to P' \quad \vdash \{P'\}\ cmd\ \{Q'\} \quad Q' \to Q}{\vdash \{P\}\ cmd\ \{Q\}}$$

Verification : Test or Proof

Test

- Requires Testability of Programs (initialisable, reproducible behaviour, sufficient control over non-determinism)
- Can be also Work-Intensive !!!
- Requires Test-Tools
- Requires a Formal Specification
- Makes Test-Hypothesis, which can be hard to justify !

Summary

Formal Proof

- Can be very hard – up to infeasible (no one will probably ever prove correctness of MS Word!)
- Proof Work typically exceeds programming work by a factor 10!
- Tools and Tool-Chains necessary

- *Makes assumptions on language, method, tool-correctness, too !*

Validation : Test or Proof (end)

Test and Proof are Complementary ...

- ... and extreme ends of a continuum : from static analysis to formal proof of "deep system properties"

- In practice, a good "verification plans" will be necessary to get the best results with a (usually limited) budget !!!
  - detect parts which are easy to test
  - detect parts which are easy to prove
  - good start: maintained formal specification

Hoare – Logic: Outlook

- Can we be sure, that the logical systems are consistent ?

  Well, yes, practically.
  (See Hales Article in AMS: "Formal Proof", 2008.
  http://www.ams.org/ams/press/hales-nots-dec08.html)

- Can we ever be sure, that a specification "means" what we intend ?

  Well, no.
  But when can we ever be entirely sure that we know what we have in mind ?
  But at least, we can gain confidence validating specs, i.e. by
  animation and test, thus, by experimenting with them ...

Hoare – Logic: Outlook

$$\frac{}{\vdash \{P \wedge \neg cond\}\ \text{WHILE } cond \text{ DO } c\ \{P \wedge \neg cond\}}$$

$$\frac{P = P' \qquad \vdash \{P'\}\ cmd\ \{Q'\} \qquad Q' = Q}{\vdash \{P\}\ cmd\ \{Q\}}$$