2025/26

Cycle Ingénieur – 2ème année
Département Informatique

Verification and Validation

Part III : Formal Specification with UML/MOAL

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Plan of the Chapter

- Syntax & Semantics of our own language

MOAL

- mathematical
  - object-oriented
  - UML-annotation
  - language

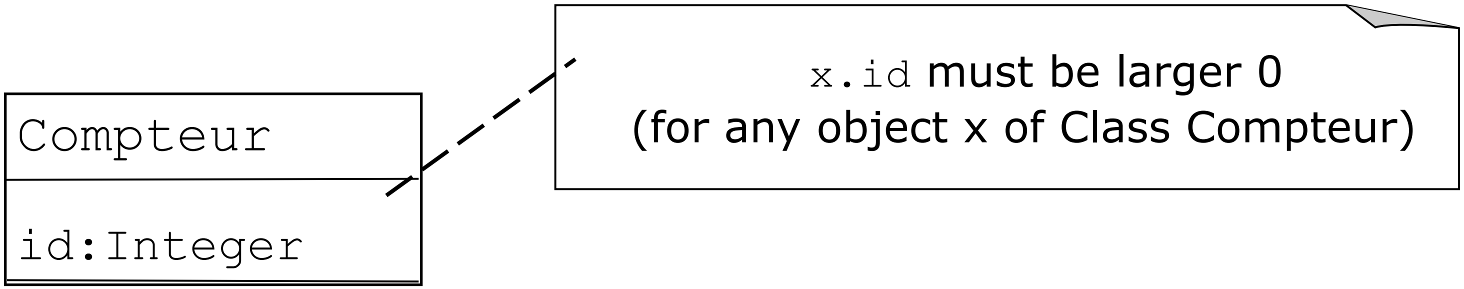
(conceived as the „essence" of annotation languages like OCL, JML, Spec#, ACSL, ...)

Plan of the Chapter

- Concepts of MOAL
  - Basis: Logic and Set-theory
  - MOAL is a Typed Language
  - Basic Types, Sets, Pairs and Lists
  - Object Types from UML
  - Navigation along UML attributes and associations (Idea from OCL and JML)
- Purpose :
  - Class Invariants
  - Method Contracts with Pre- and Post-Conditions
  - Annotated Sequence Diagrams for Scenarios, . . .

Motivation: Why Logical Annotations

- More precision needed
  (like JML, VCC) that constrains an underlying state σ



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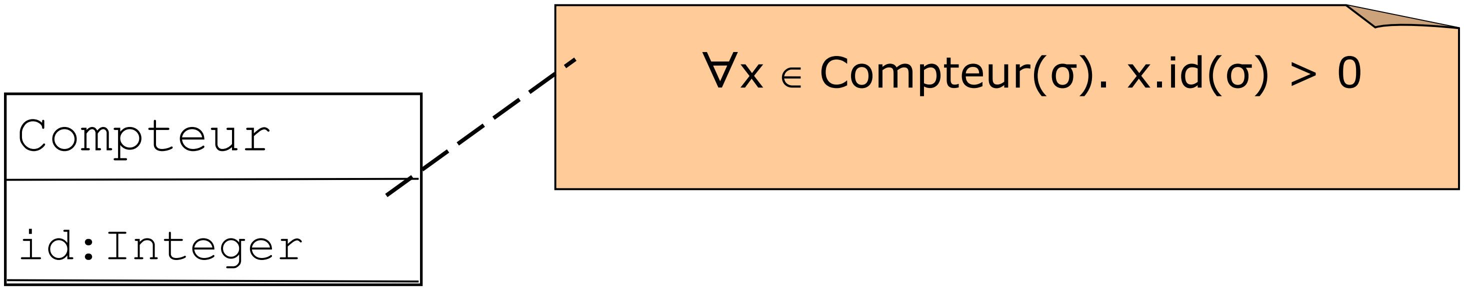

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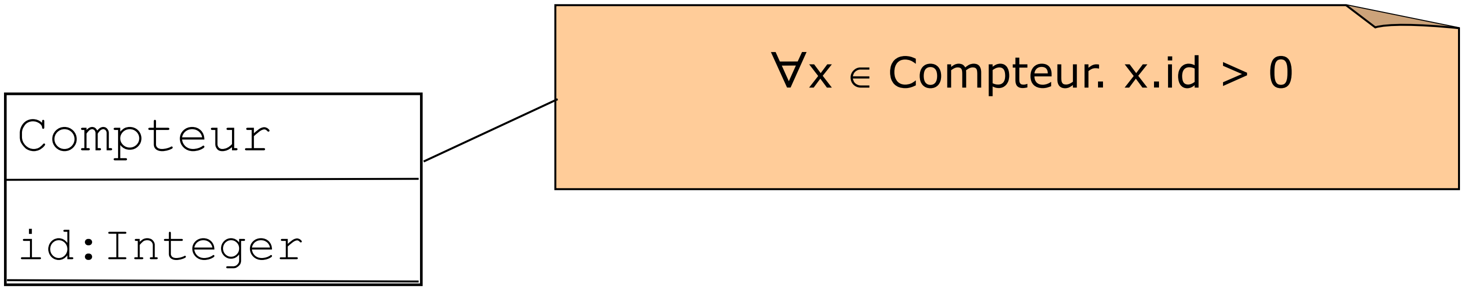

... by abbreviation convention if no confusion arises.

Motivation: Why Logical Annotations

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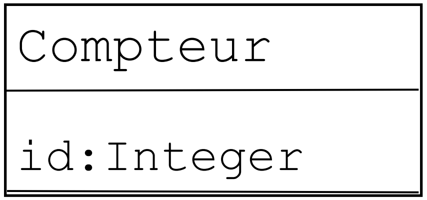


$$\text{definition } inv_{Compteur}(\sigma) \equiv \forall x \in Compteur(\sigma).\ x.id(\sigma) > 0$$

... or by convention

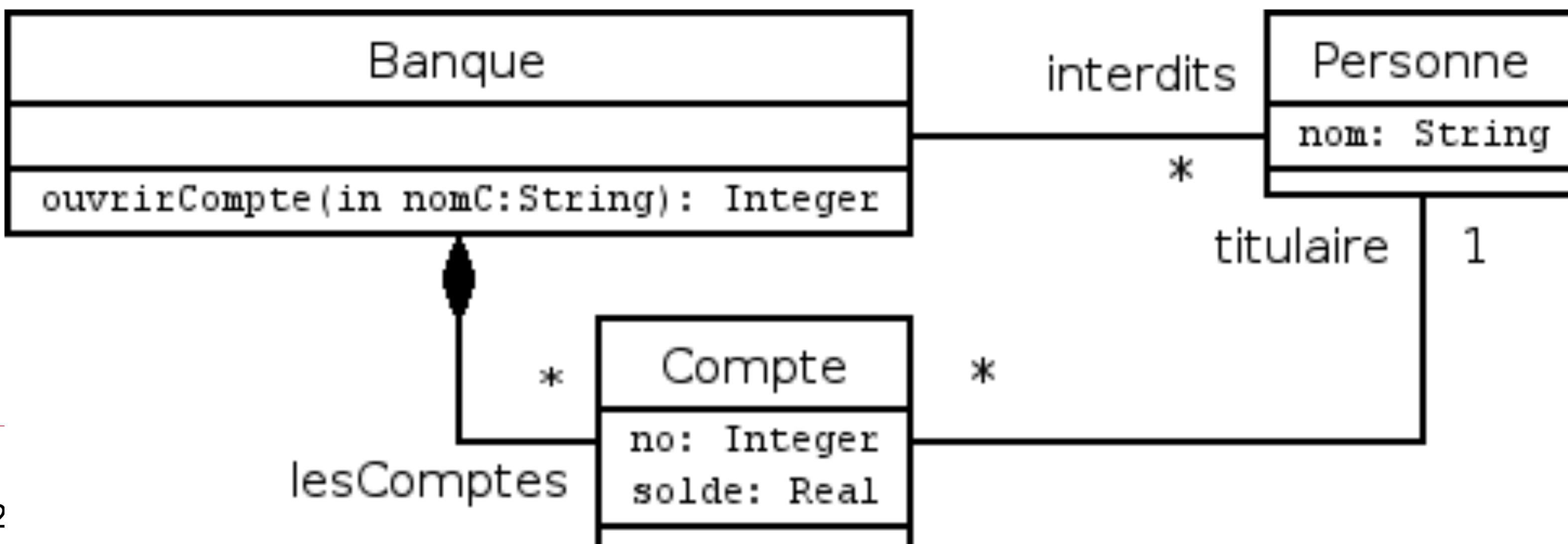
$$\text{definition } inv_{Compteur} \equiv \forall x \in Compteur.\ x.id > 0$$

... or as mathematical definition in a separate document

A first Glance to an Example: Bank

Opening a bank account. Constraints:

- there is a blacklist
- no more overdraft than 200 EUR
- there is a present of 15 euros in the initial account
- account numbers must be distinct.



A first Glance to an Example: Bank (2)

```
definition unique ≡ isUnique(.no)(Compte)
definition noOverdraft ≡ ∀c ∈ Compte. c.id ≥ -200

definition pre_ouvrirCompte(b:Banque, nomC:String)≡
                                ∀p ∈ Personne. p.nom ≠ nomC

definition post_ouvrirCompte(b:Banque,nomC:String,r::Int)≡
        |{p ∈ Personne | p.nom = nomC ∧ isNew(p)}| = 1
      ∧ |{c∈Compte | c.titulaire.nom = nomC}|  = 1
      ∧ ∀c∈Compte. c.titulaire.nom = nomC
             ⟶ c.solde = 15 ∧ isNew(c)
      …
```

MOAL: a specification langage?

- In the following, we will discuss the MOAL Language in more detail ...

Syntax and Semantics of MOAL

- The usual logical language:

  - True, False
  - negation : $\neg E$,
  - or: $E \vee E'$, and: $E \wedge E'$, implies: $E \longrightarrow E'$
  - $E = E'$, $E \neq E'$,
  - if C then E else E' endif
  - let x = E in E'

  - Quantifiers on sets and lists:

$\forall x \in Set.\ P(x)$ $\exists x \in Set.\ P(x)$

Syntax and Semantics of MOAL

- MOAL is (like OCL or JML) a typed language.
  - Basic Types:

    Boolean, Integer, Real, String
  - Pairs:

    $X \times Y$
  - Lists:

    List(X)
  - Sets:

    Set(X)

Syntax and Semantics of MOAL

- The arithmetic core language.

  expressions of type `Integer` or `Real`:

  - `1,2,3 ...   resp. 1.0, 2.3, pi.`
  - `- E, E + E',`
  - `E * E', E / E',`
  - `abs(E), E div E', E mod E'...`

Syntax and Semantics of MOAL

- The expressions of type `String`:
  - *S* concat S'
  - *size(S)*
  - substring(i,j,S)
  - 'Hello'

Syntax and Semantics of MOAL Sets

- $| S |$ size as Integer
- $isUnique(f)(S) \equiv \forall x,y \in S.\ f(x)=f(y) \longrightarrow x=y$
- $\{\}$, $\{a,b,c\}$ *empty and finite sets*
- $e \in S$, $e \notin S$ is element, not element
- $S \subseteq S'$ is subset
- $\{x \in S \mid P(x)\}$ filter
- $S \cup S'$, $S \cap S'$ union, intersection between sets of same type
- Integer, Real, String ... are symbols for the set of all Integers, Reals,

Syntax and Semantics of MOAL Pairs

- `(X,Y)` pairing
- `fst(X,Y) = X` projection
- `snd(X,Y) = Y` projection

Syntax and Semantics of MOAL Lists

Lists *S* have the following operations:

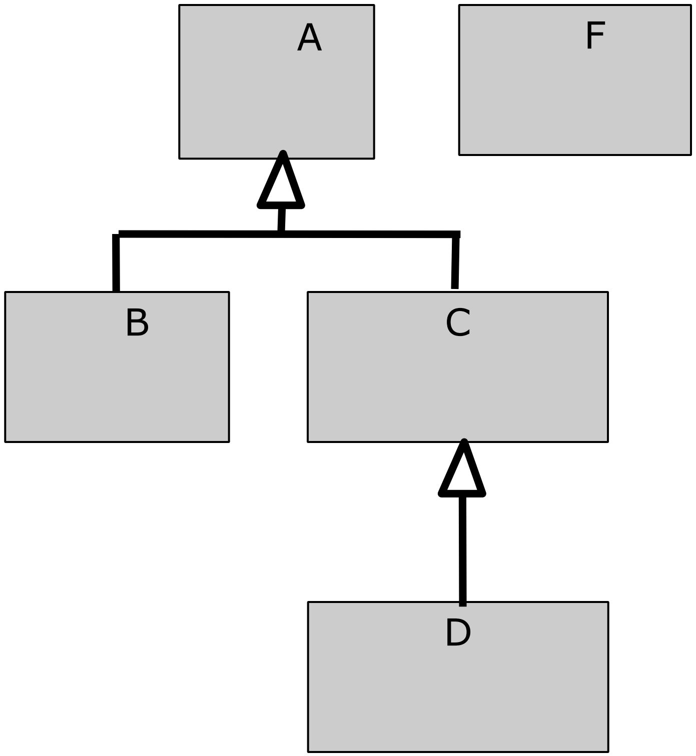
- `x ∈ L` -- is element (overload!)
- `|S|` -- length as Integer
- `head(L),last(L)`
- *nth*`(L,`*i*`)` -- for `i` between `0` et `|S|-1`
- *L@L'* -- concatenate
- *e#S* -- append at the beginning
- `∀x∈List. P(x)` -- quantifiers :
- `[x∈L | P(x)]` -- filter
- `[1,2,3]` -- denotations of lists

Syntax and Semantics of Objects

- Objects and Classes follow the semantics of UML
  - inheritance / subtyping
  - casting
  - objects have an id
  - NULL is a possible value in each class-type
  - for any class A, we assume a function:

    $$A(\sigma)$$

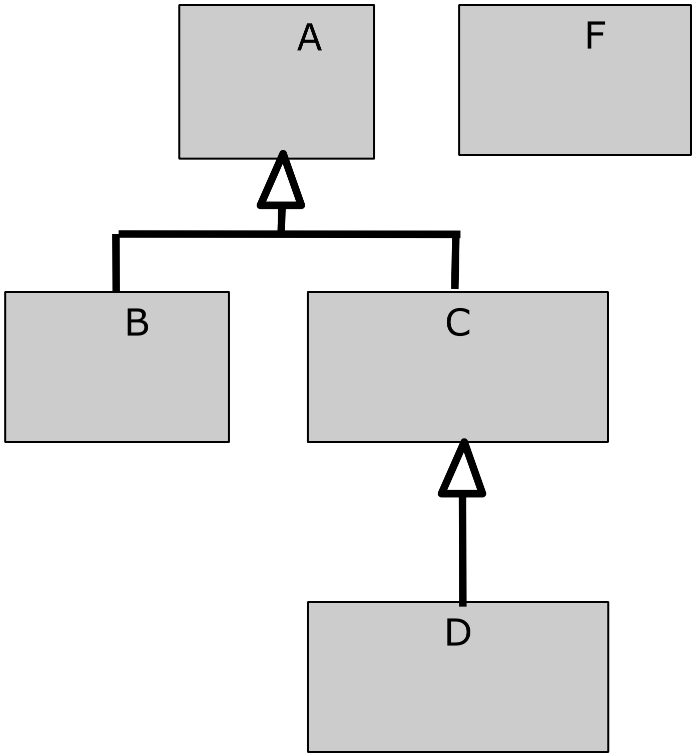
    which returns the set of instances of class A in state σ



Syntax and Semantics of Objects

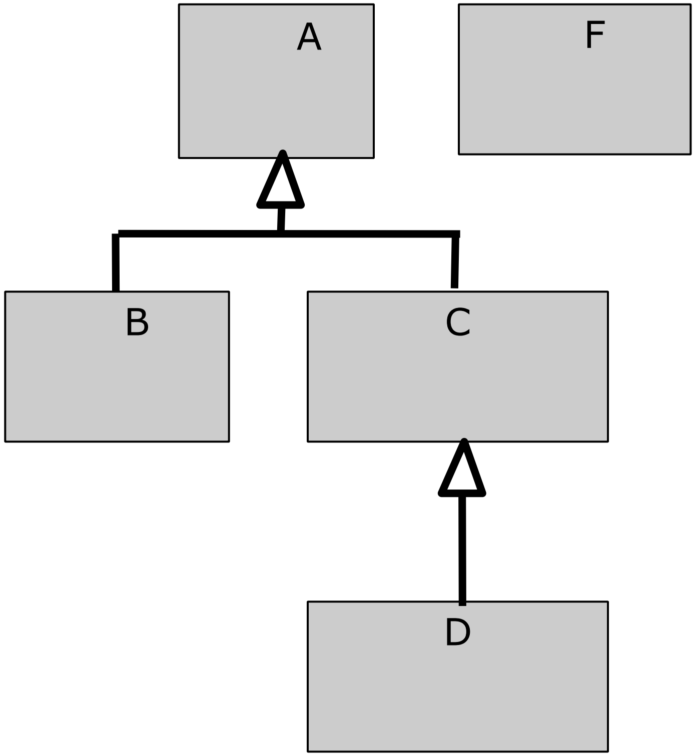
- Objects and Classes follow the semantics of UML

  Recall that we will drop the index (σ) whenever it is clear from the context



Syntax and Semantics of Objects

- As in all typed object-oriented languages casting allows for converting objects.
- Objects have two types:
  - the « apparent type » (also called static type)
  - the « actual type » (the type at creation)
  - casting changes the apparent type along the class hierarchy, but not the actual type



Syntax and Semantics of Objects

- Assume the creation of objects

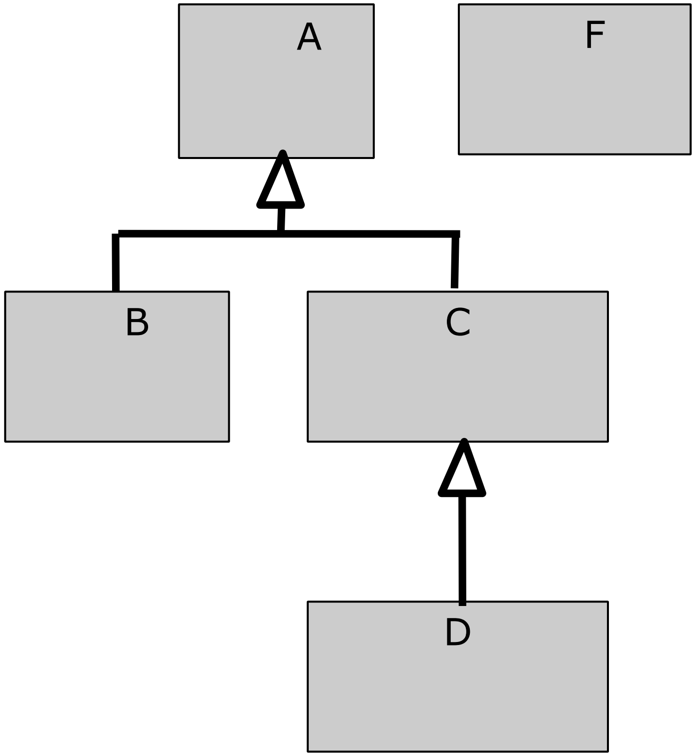
  `a in class A,b in class B,`
  `c in class C,d in class D,`

- Then casting:

  `⟨F⟩b is illtyped`

  `⟨A⟩b has apparent type A,`
  `    but actual type B`

  `⟨A⟩d has apparent type A,`
  `    but actual type D`



Syntax and Semantics of OCL / UML

- We will also apply cast-operators to an entire set: So

  `⟨A⟩B(σ) (or just: ⟨A⟩B)`

  - is the set of instances of B casted to A.

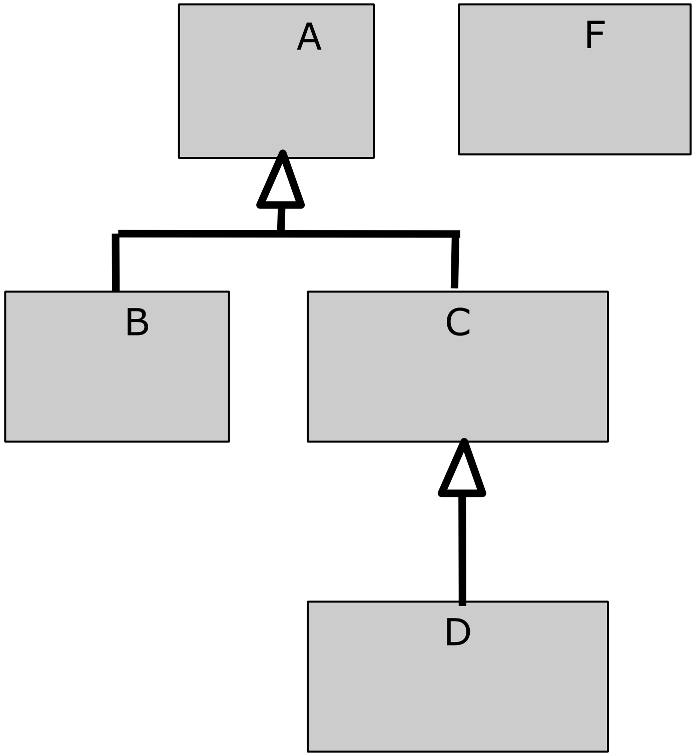
  - We have:

    $\langle A\rangle B \cup \langle A\rangle C \subseteq A$

    but:

    $\langle A\rangle B \cap \langle A\rangle C = \{\}$

    and also: $\langle A\rangle D \subseteq A$ (for all states σ)



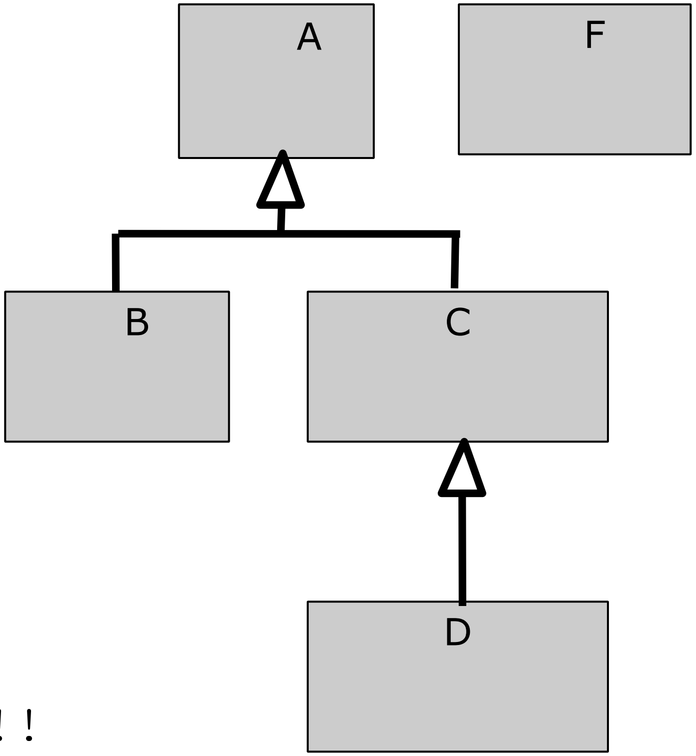
Syntax and Semantics of Objects

- Instance sets can be used to determine the actual type of an object:

  `b` ∈ `B`

  corresponds to Java's `instanceof` or `OCL's isKindOf`. Note that casting does NOT change the actual type:

  ⟨`A`⟩`b` ∈ `B`, and ⟨`B`⟩⟨`A`⟩`b = b !!!`



Syntax and Semantics of Objects

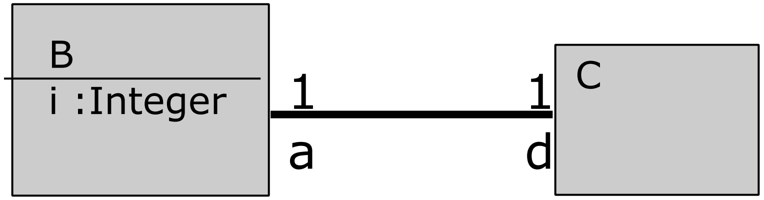
- Summary:
  - there is the concept of **actual** and **apparent** type (anywhere outside of Java: **dynamic** and **static** type)
  - type tests check the former
  - type casts influence the latter, but not the former
  - up-casts possible
  - down-casts invalid
  - consequence: up-down casts are identities.

Syntax and Semantics of Object Attributes

- Objects represent structured, typed memory in a state σ. They have attributes.

  Attributes can have class types.

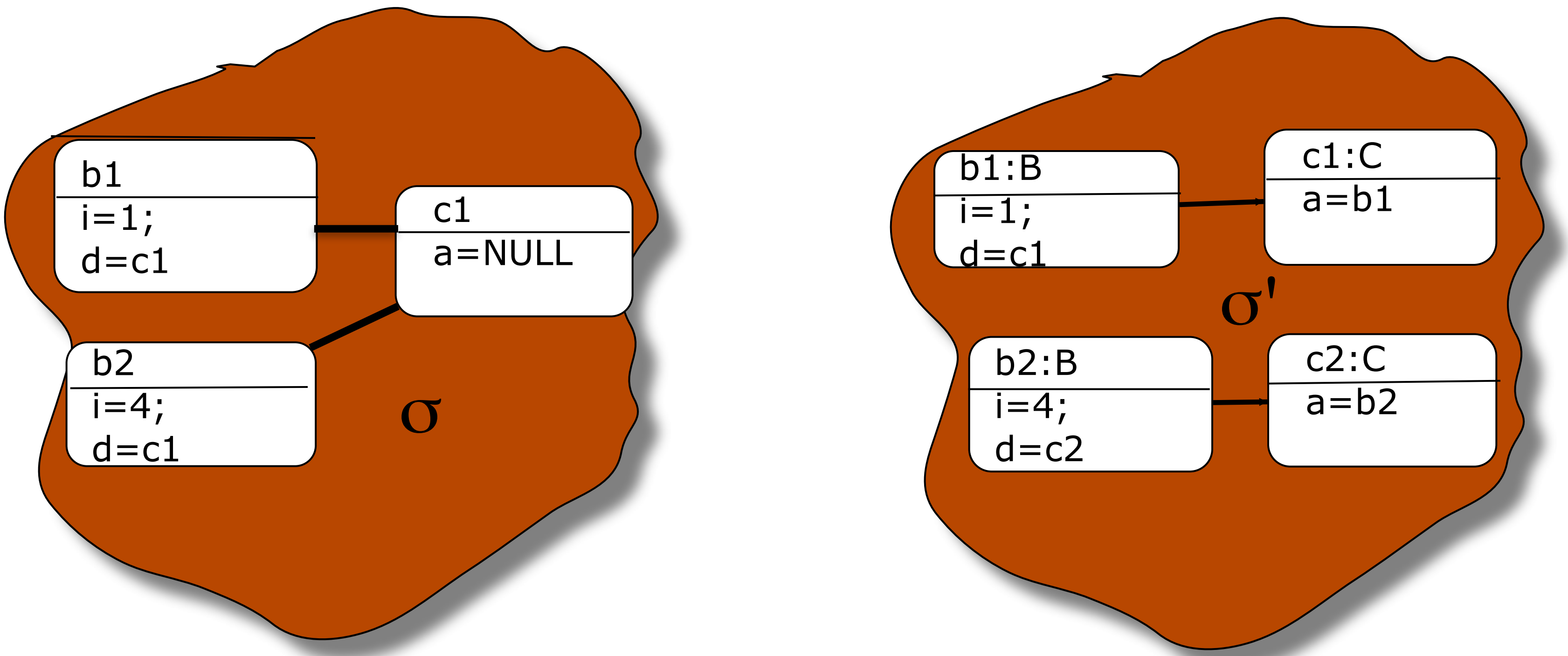
- Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)



Syntax and Semantics of Object Attributes

- Example:

  attributes of class type in states σ' and σ.



Syntax and Semantics of Object Attributes

- each attribute is represen-
ted by a function in *MOAL*.
The class diagram right corresponds to delaration of accessor functions:

  .i(σ) :: B -> Integer

  .a(σ) :: C -> B

  .d(σ) :: B -> C

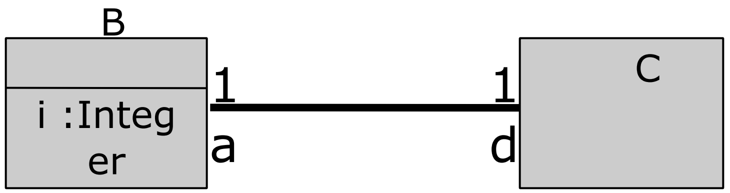
- Applying the σ–convention, this makes navigation expressions possible:
  - ```
    b1.d :: C
    c1.a :: B                b1.d.a.d.a ...
    ```
    ```

Syntax and Semantics of Object Attributes

- Object assessor functions are „dereferentiations of pointers in a state“
- Accessor functions of class type are strict wrt. NULL.
  - `NULL.d = NULL`
    `NULL.a = NULL`
  - Note that navigation expressions depend on their underlying state:
    b1.d(σ).a(σ).d(σ).a(σ) = NULL
    b1.d(σ’).a(σ’).d(σ’).a(σ’) = b1 !!!

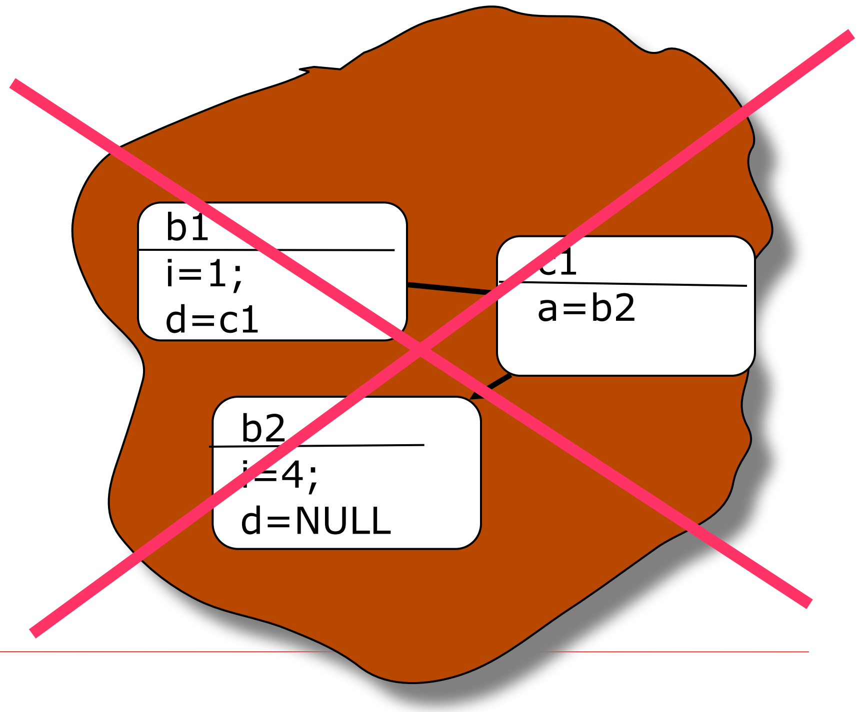
(cf. Object Diagram pp 27)

Syntax and Semantics of Object Attributes

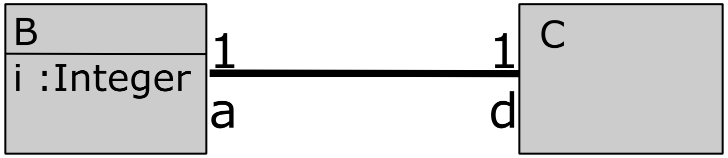


- Note that associations are meant to be « relations » in the mathematical sense.

Thus, states (object-graphs) of this form do not represent the 1:1 association:



Syntax and Semantics of Object Attributes



- This is reflected by 2 « association integrity constraints ».
  For the 1-1-case, they are:

> ➢ definition $ass_{B.d.a} \equiv \forall x \in B.\ x.d.a = x$
>
> ➢ definition $ass_{C.a.d} \equiv \forall x \in C.\ x.a.d = x$

Syntax and Semantics of Object Attributes

- Object assessor functions are „dereferentiations of pointers in a state"
- Accessor functions of class type are strict wrt. NULL.
  - ```
    NULL.d = NULL
    NULL.a = NULL
    ```
  - Note that navigation expressions depend on their underlying state:

    b1.d(σ) .a(σ) .d(σ) .a(σ) = NULL

    b1.d(σ').a(σ').d(σ').a(σ') = b1 !!!

    (cf. Object Diagram pp 28)
    ```

Syntax and Semantics of Object Attributes

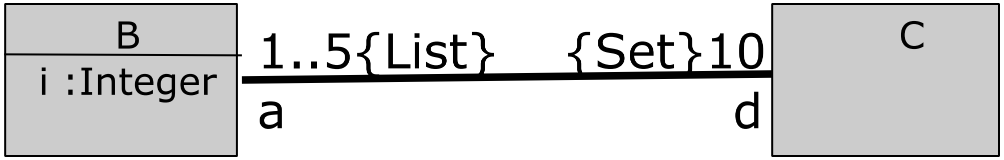
- Attibutes can be List or Sets of class types:

- Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)

- In analysis-level Class Diagrams, the type information is still omitted; due to overloading of $\forall x \in X .\ P(x)$ etc. this will not hamper us to specify …

Syntax and Semantics of Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:



> - definition $card_{B.d} \equiv \forall x \in B.\ |x.d| = 10$
> - definition $card_{C.a} \equiv \forall x \in C.\ 1 \leq |x.a| \leq 5$

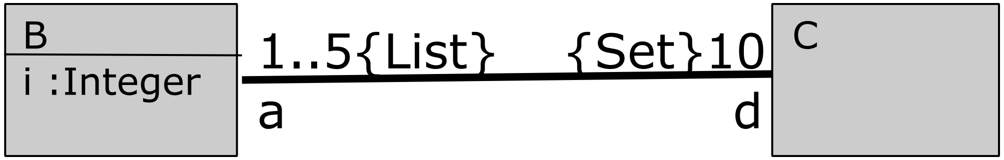
Syntax and Semantics of Object Attributes

- Accessor functions are defined as follows for the case of NULL:

  - NULL.d = {}   -- mapping to the neutral element
  - NULL.a = []   -- mapping to the neural element.

Syntax and Semantics of Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:



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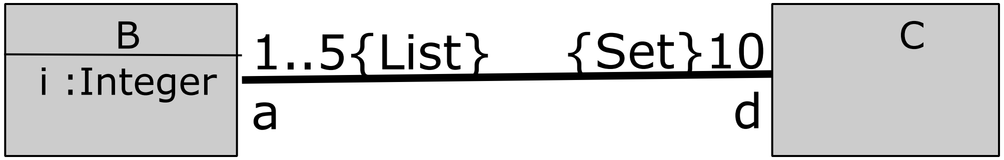
- ➢ definition $card_{C.a} \equiv \forall x \in C.\ 1 \leq |x.a| \leq 5$

Syntax and Semantics of Object Attributes

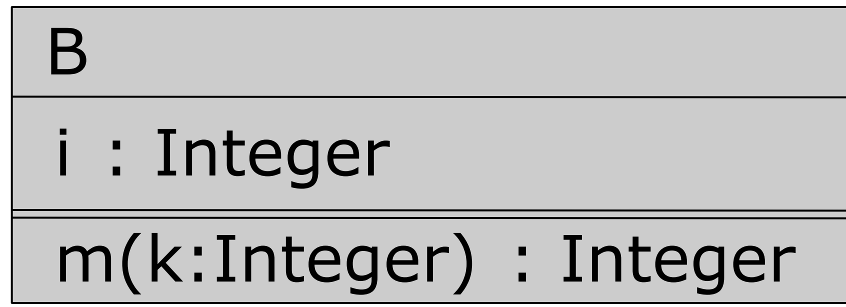
- The corresponding association integrity constraints for the *-*-case are:



- $\text{definition ass}_{B.d.a} \equiv \forall x \in B.\ x \in x.d.a$
- $\text{definition ass}_{C.a.d} \equiv \forall x \in C.\ x \in x.a.d$

Operations in UML and MOAL

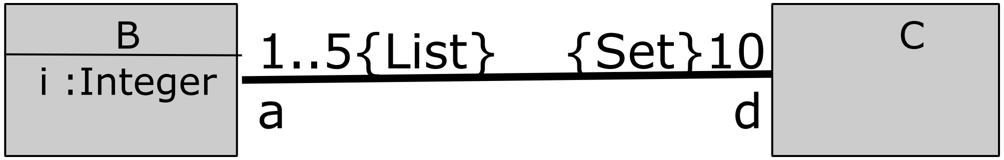
- Many UML diagrams talk over a sequence of states (not just individual global states)
- This appears for the first time in so-called contracts for (Class-model) methods:



- The « method » m can be seen as a « transaction » of a B object transforming the underlying pre-state σ_{pre} in the state « after » m yielding a post-state σ.

Syntax and Semantics of Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:



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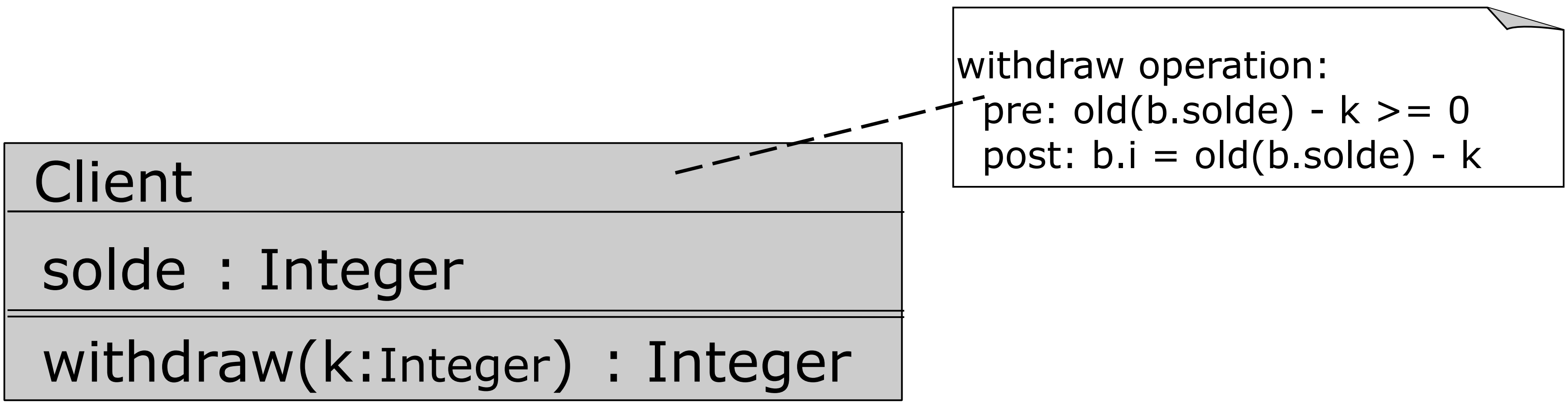
- ➢ definition $card_{C.a} \equiv \forall x \in C.\ 1 \leq |x.a| \leq 5$

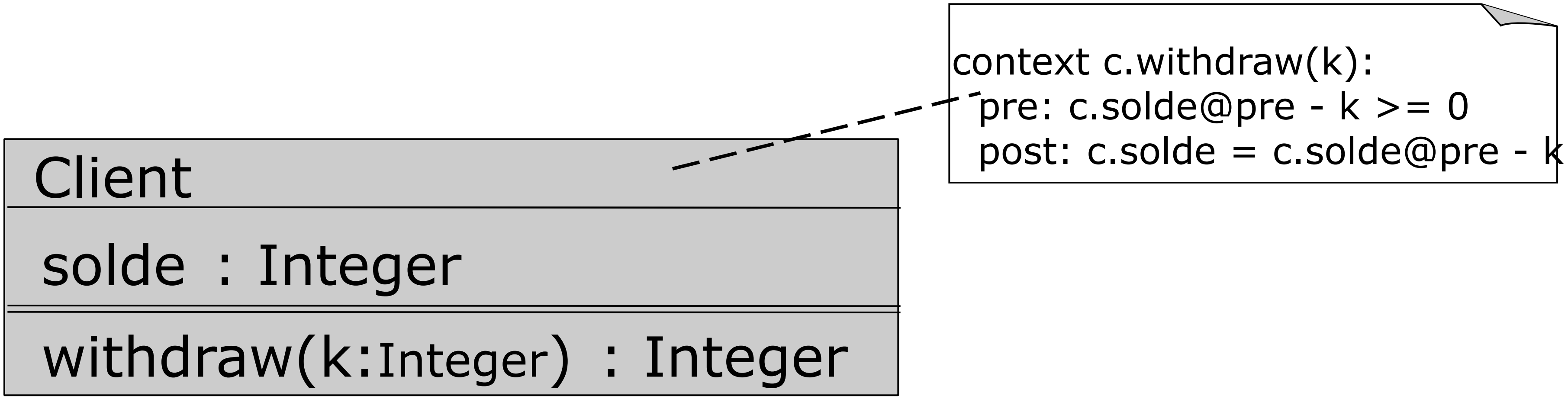
Operations in UML and MOAL

- Syntactically, contracts are annotated like this (JML-ish):



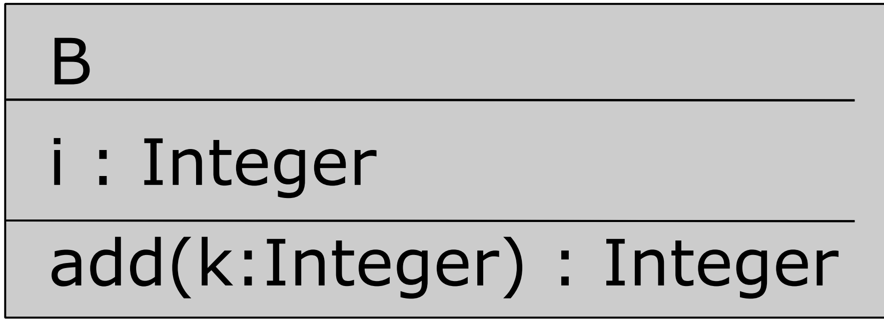
Operations in UML and MOAL

- ... or like this (OCL-ish):



Operations in UML and MOAL Contracts

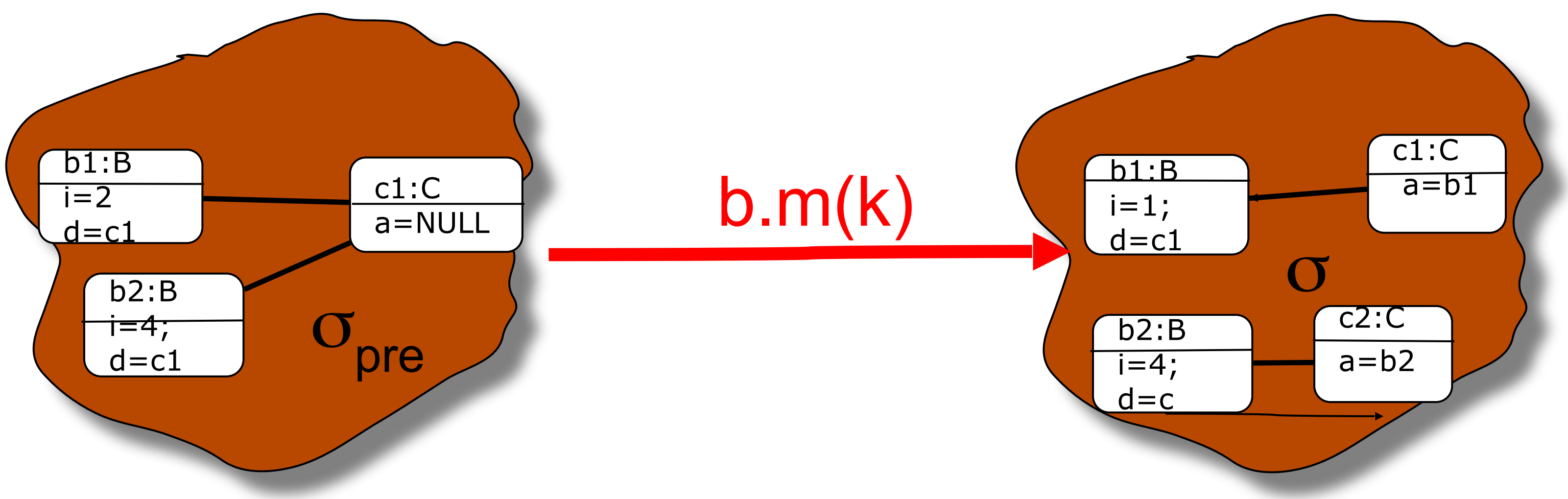
- This appears for the first time in so-called contracts for (Class-model) methods:



- The « method » add can be seen as a « transaction » of a B object transforming the underlying pre-state σ_{pre} in the state « after » add yielding a post-state σ.

Syntax and Semantics of MOAL Contracts

- Again: This is the view of a transaction (like in a data-base), it completely abstracts away intermediate states or time. (This possible in other models/calculi, like the Hoare-calculus, though).



Syntax and Semantics of MOAL Contracts

- Consequence:
  - The pre-condition is a formula referring to the σ_{pre} and the method arguments b1, a_1, ..., a_n only.
  - the post-condition is only assured if the pre-condition is satisfied
  - otherwise the method
    - ...may do anything on the state and the result,
      may even behave correctly , may non-terminate !
    - raise an exception
      (recommended in Java Programmer Guides
      for public methods to increase robustness)

Syntax and Semantics of MOAL Contracts

- Consequence:
  - The post-condition is a formula referring to both σ_{pre} and σ, the method arguments b1, a_1, ..., a_n and the return value captured by the variable result.
  - any transition is permitted that satisfies the post-condition (provided that the pre-condition is true)

Syntax and Semantics of MOAL Contracts

- Consequence:
  - The semantics of a method call:

$$b1.m(a_1, ..., a_n)$$

is thus:

$$pre_m(b1,a_1, ..., a_n)(\sigma_{pre})$$

$$\longrightarrow$$

$$post_m(b1,a_1, ..., a_n,result)(\sigma_{pre},\sigma)$$

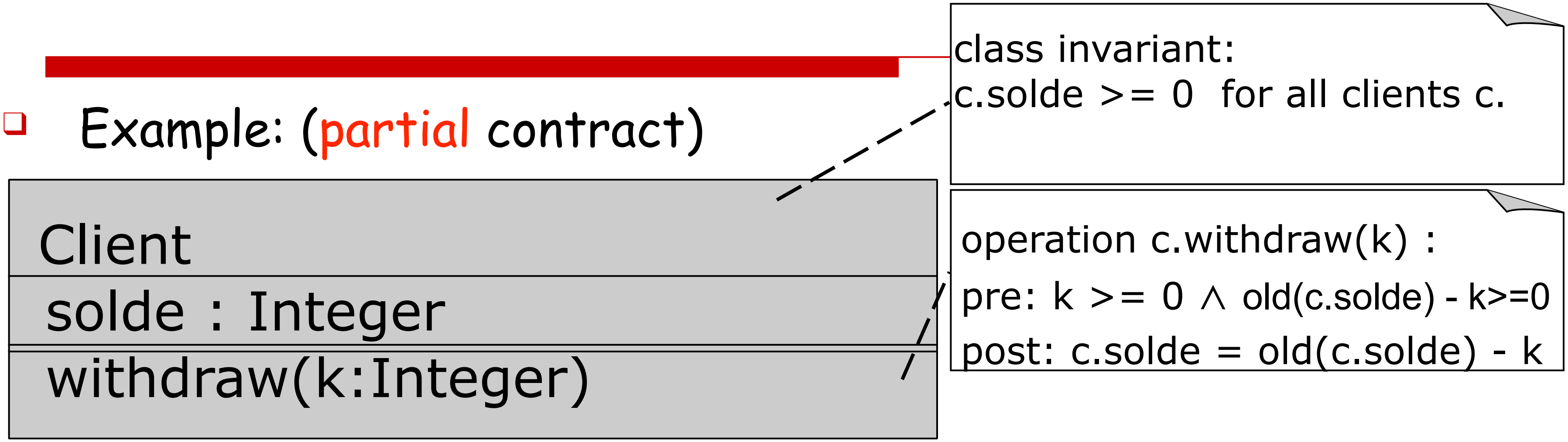
  - Note that moreover all global class invarants have to be added for both pre-state σ_{pre} and post-state σ !

For an entire transition, the following must hold:

$$Inv(\sigma_{pre}) \wedge pre_m(b1,a_1, ..., a_n)(\sigma_{pre}) \wedge post(b1,a_1, ..., a_n,result)(\sigma_{pre},\sigma) \wedge Inv(\sigma)$$

Syntax and Semantics of MOAL Contracts

- Example: (partial contract)



- definition $inv_{Client}(\sigma) \equiv$
  $\forall c \in Client(\sigma).\ 0 \leq c.solde(\sigma)$
- definition $pre_{withdraw}(c, k)(\sigma) \equiv$
  $0 \leq k \wedge 0 \leq c.solde(\sigma) - k$
- definition $post_{withdraw}(c, k, result)(\sigma_{pre}, \sigma) \equiv$
  $c.solde(\sigma) = c.solde(\sigma_{pre}) - k$

Syntax and Semantics of MOAL Contracts

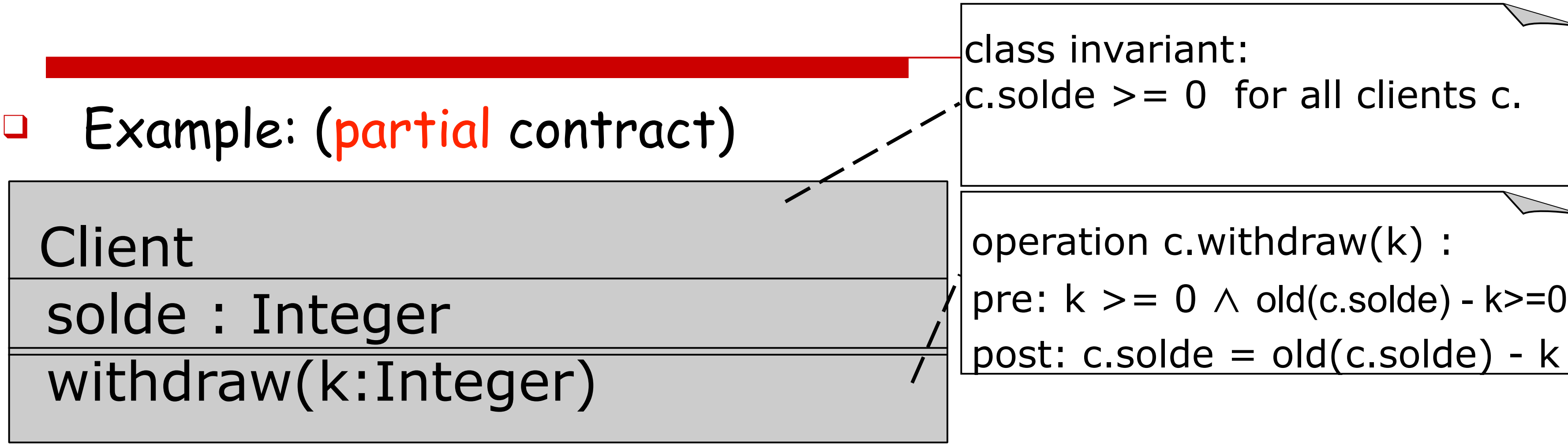
- Notation (which we call : σ-convention):
  - In order to relax notation, we will drop the σ and use for applications to σ_{pre} the old-notation:

Client(σ)	becomes	Client
Client(σ_{pre})	becomes	old(Client)
`c.solde`(σ_{pre})	becomes	old(`c.solde`)

etc.

Syntax and Semantics of MOAL Contracts

- Example: (partial contract)



- definition $inv_{Client} \equiv$
  $\forall c \in Client.\ 0 \leq c.solde$
- definition $pre_{withdraw}(c, k)(\sigma) \equiv$
  $0 \leq k \wedge 0 \leq c.solde - k$
- definition $post_{withdraw}(c, k, result) \equiv$
  $c.solde = old(c.solde) - k$

σ - convention

Syntax and Semantics of MOAL Contracts

- Example (total contract):

class invariant:
c.solde >= 0 for all clients c.

Client
solde : Integer
withdraw(k:Integer) : {ok,nok}

operation c.withdraw(k) :
pre: True
post: if k >= 0 ∧ old(c.solde)>=k
then c.solde = old(c.solde)-k ∧
result = ok
else c.solde = old(c.solde) ∧
result = nok

- definition $inv_{Client} \equiv \forall c \in Client.\ 0 \leq c.solde$
- definition $pre_{withdraw}(c, k)(\sigma_{pre}) \equiv True$
- definition $post_{withdraw}(c, k, result)(\sigma_{pre}, \sigma) \equiv$

$$\begin{array}{l} \text{if } 0 \leq k \wedge k \leq c.solde(\sigma_{pre}) \\ \text{then } c.solde(\sigma) = c.solde(\sigma_{pre}) - k \wedge result = ok \\ \text{else } c.solde(\sigma) = c.solde(\sigma_{pre}) \wedge result = nok \end{array}$$

Syntax and Semantics of MOAL Contracts

- Example (total contract):

Client
solde : Integer
withdraw(k:Integer) : {ok,nok}

class invariant:
c.solde >= 0 for all clients c.

operation c.withdraw(k) :
pre: True
post: if k >= 0 ∧ old(c.solde)>=k
then c.solde = old(c.solde)-k ∧
result = ok
else c.solde = old(c.solde) ∧
result = nok

- definition inv_{Client} ≡ ∀c∈Client . 0≤c.solde
- definition $pre_{withdraw}$(c, k) ≡ True
- definition $post_{withdraw}$(c, k, result) ≡
  if 0≤k ∧ k ≤ old(c.solde)
  then c.solde = old(c.solde) - k ∧ result = ok
  else c.solde = old(c.solde) ∧ result = nok

σ - convention

Semantics of MOAL Contracts

- Two predicates are helpful when defining contracts. They exceptionally refer to both (σ_{pre},σ)

  - `isNew(p)` (σ_{pre},σ) is true only if object p of class C does not exist in σ_{pre} but exists in σ

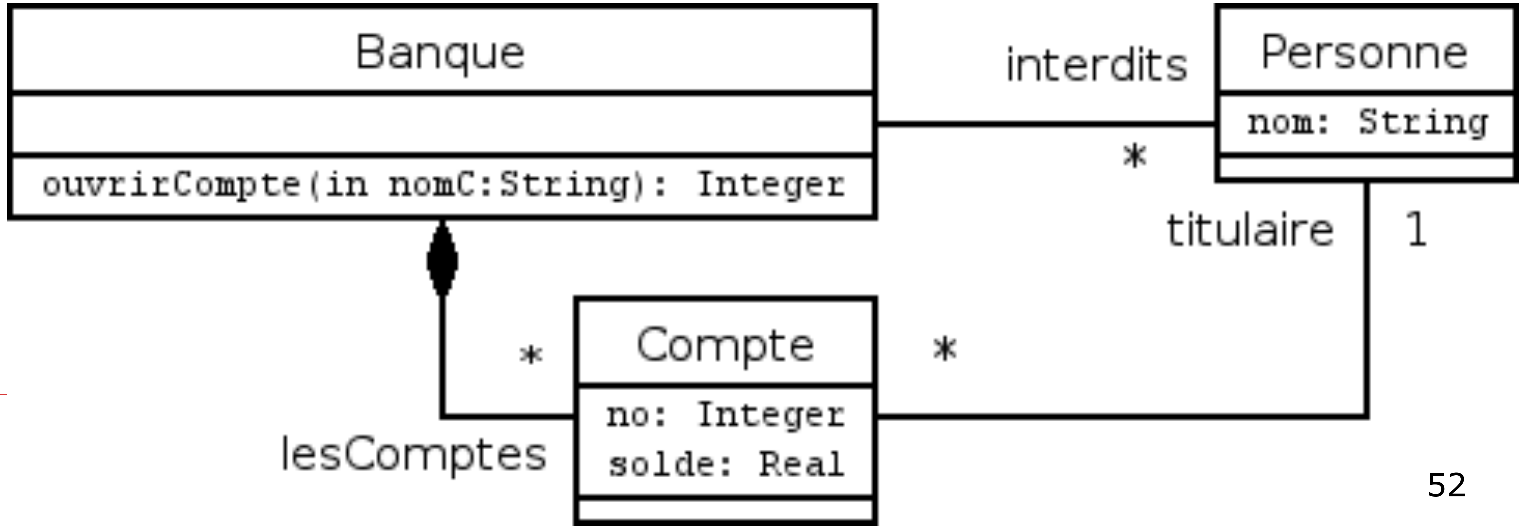
  - modifiesOnly(S)(σ_{pre},σ) is only true iff
    - all objects in σ_{pre} are except those in S identical in σ
    - all objects in σ exist either in are or are contained in S

    With this predicate, one can express : „and nothing else changes". It is also called «framing condition».

A Revision of the Example: Bank

Opening a bank account. Constraints:

- there is a blacklist
- no more overdraft than 200 EUR
- there is a present of 15 euros in the initial account
- account numbers must be distinct.



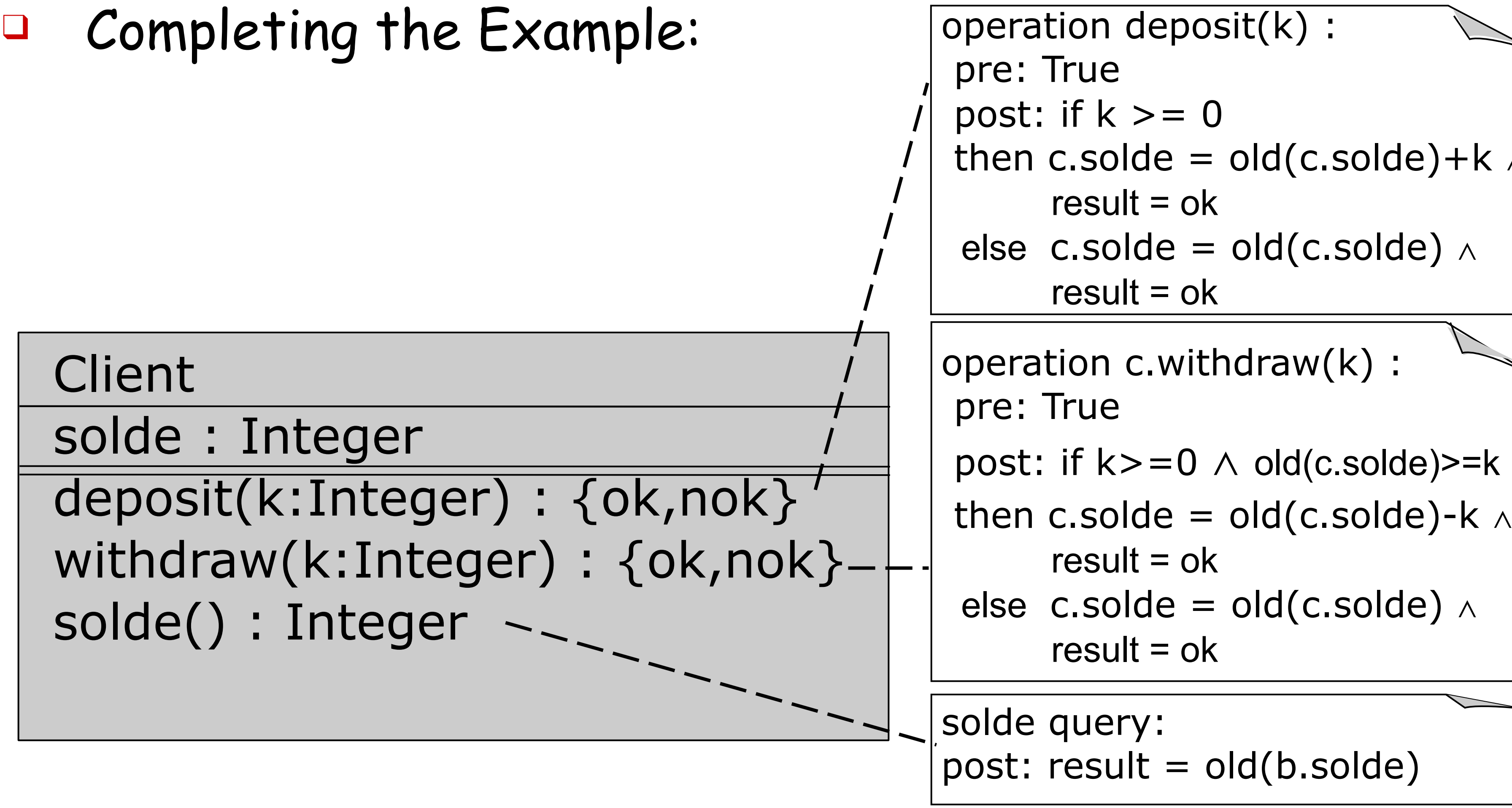
A Revision of the Example: Bank (2)

```
definition pre_ouvrirCompte(b:Banque, nomC:String)≡
                          ∀p ∈ Personne. p.nom ≠ nomC

definition post_ouvrirCompte(b:Banque,nomC:String,r:Integer)≡
     |{p ∈ Personne | p.nom = nomC}| = 1
     ∧  ∀p∈Personne. p.nom = nomC ⟶ isNew(p)
     ∧ |{c∈Compte | c.titulaire.nom = nomC}| = 1
     ∧ ∀c∈Compte. c.titulaire.nom = nomC ⟶ c.solde = 15
                                             ∧ isNew(c)
     ∧ b.lesComptes= old(b.lesComptes)
                    ∪ {c∈Compte | c.titulaire.nom = nomC}
     ∧ b.interdits =old(b.interdits)
                    ∪ {p ∈ Personne | p.nom = nomC}
     ∧ modifiesOnly({b}∪{c∈Compte | c.titulaire.nom = nomC}
                    ∪ {p ∈ Personne | p.nom = nomC})
```

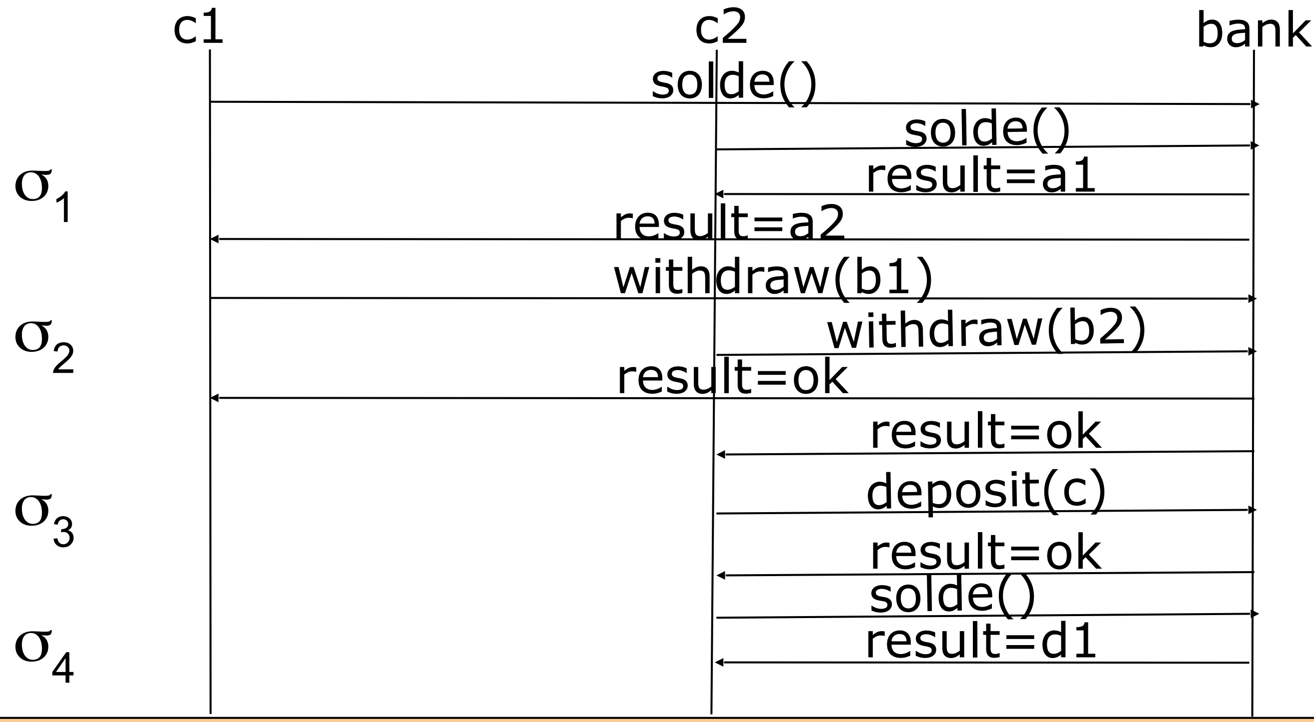
Operations in UML and MOAL

- Completing the Example:



Operations in UML and MOAL

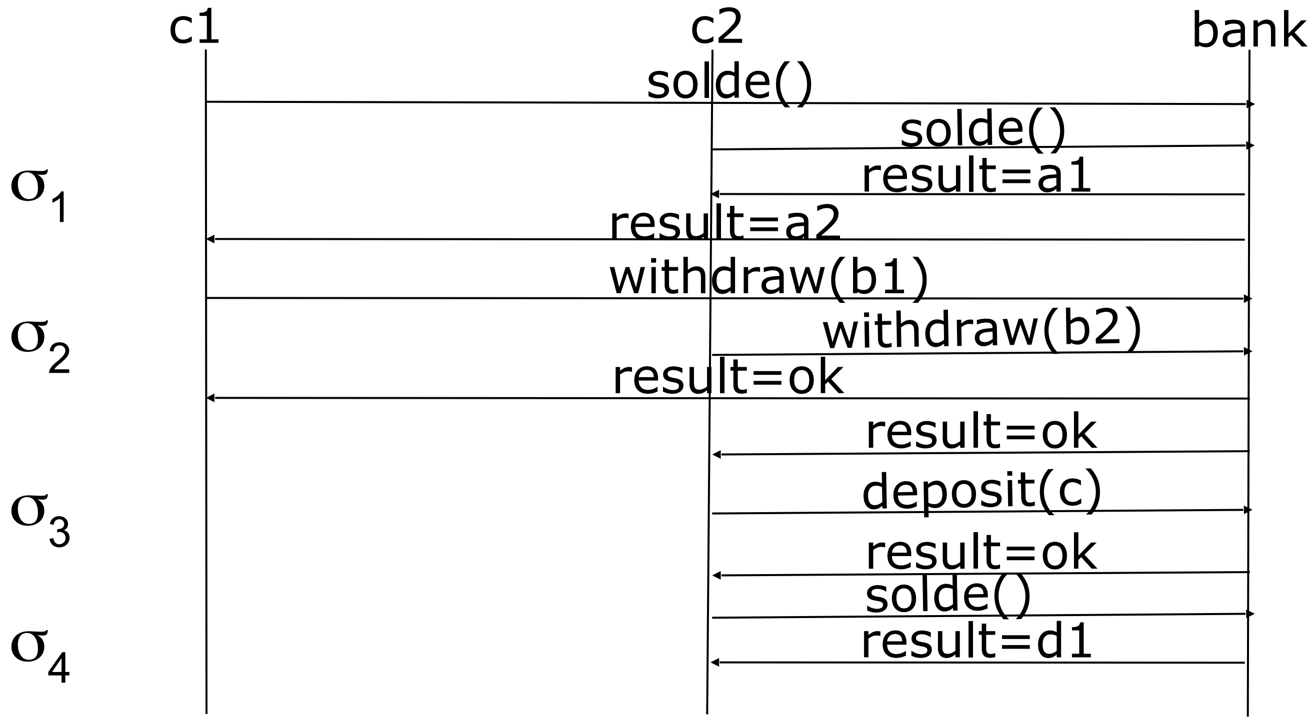
- Abstract Concurrent Test Scenario:



assert $c1.solde(\sigma_4)=a2-b1 \wedge b1 \geq 0 \wedge a2 \geq b1$

Operations in UML and MOAL

- Abstract Concurrent Test Scenario:



Any instance of b1 and a1 is a test ! This is a „Test Schema" !
Note: b1 can be chosen dynamically during the test !

Summary

- MOAL makes the UML to a real, formal specification language
- MOAL can be used to annotate Class Models, Sequence Diagrams and State Machines
- Working out, making explicit the constraints of these Diagrams is an important technique in the transition from Analysis documents to Designs.