université PARIS-SACLAY | FACULTÉ DES SCIENCES D'ORSAY

semaine du 30 janvier février 2023

Automates et Applications
M1–MPRI Orsay

TD n° 4

1 Büchi automata and ω-regular languages

1. Give a Büchi automaton which accepts words of the form : $(ba)^*(ab)^\omega$ over the alphabet $\{a, b\}$
2. Give a Büchi automaton (non-deterministic) over $\Sigma = \{a, b, c\}$ that recognizes all infinite words containing a pair of letters a, b separated by 4 occurrence of c.
3. Give a sufficient condition for a non empty language L such that $\epsilon \notin L$ to be such that L^ω is finite.
4. Let L be an ω-regular language and A a regular language (of finite words). Consider

$$L' = \bigcup_{u \in A} \{w \mid uw \in L\}$$

   Is L' ω-regular?
5. Show that the singleton language $1\,01\,001\,0001\,00001\,\ldots\,0^n1\,\ldots$ is not Büchi recognizable

2 LTL

1. For every question below, we fix the set AP. Express the given condition as an LTL formula.
   (a) $AP = \{\text{req}, \text{ans}\}$ a request(req) is always followed by an answer at some point (ans)
   (b) $AP = \{\text{door}, \text{code}\}$ Every time we give the code, the door opens (immediately after)
   (c) $AP = \{\text{rouge}, \text{orange}, \text{vert}\}$ When the light is green, it will become orange and then red, not necessarily immediately after.
2. **ATTENTION** in this section w^i does not represent $wwwww\ldots$ repeated i times, but rather the suffix of w starting at position i.
   Show the following equivalences
   — $\mathcal{F}\mathcal{F}\phi \equiv \mathcal{F}\phi$
   — $\mathcal{F}\phi \equiv \phi \vee \mathcal{X}\mathcal{F}\phi$.
   — $\mathcal{X}(\phi \mathcal{U} \psi) \equiv (\mathcal{X}\phi)\mathcal{U}(\mathcal{X}\psi)$