FACULTÉ DES SCIENCES D'ORSAY

université PARIS-SACLAY

Automates et Applications
M1–MPRI Orsay

TD nº 3

1 Logic for trees

1. Define the following concepts using MSO (sometime second-order quantification is not necessary). We assume a fixed alphabet $\Sigma = \{f^2, g^2, a, b\}$.

   (a) A predicate $root(x)$ which is true if and only if x is the root of the tree

   (b) The language of trees with only internal path labelled f and leaves labelled a.

   (c) The language where every path contains at least two f.

   (d) The language where all trees have exactly one a in their leaves a.

   It can be useful to define "macros" for some auxiliary predicates such as the descendants of a node

2. For questions (b), (c), (d) of exercise 1, give a tree automaton that recognizes the language, make it TD deterministic if possible.

3. Consider the ranked alphabet $\{a^2, b^2, c^2, d^2, \#\}$. Give predicates for the following :

   (a) $P(x, y)$: there exists a path between x and y going through labels a, b, c, d in that order (i.e.there are four "nodes" between x and y)

   (b) $Q(x, Y)$: Y reachable downward from x with a path as described in question (a)

   (c) $R(x, y)$: for a given x, there exists a unique y for which the property of question (a) holds

4. The document order relation $x \preceq y$ is a builtin predicat of FO. Show that it can be expressed in MSO directly.

5. Explain how to express a DTD as a formula in MSO.