

# XPath Whole Query Optimization

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Fast query evaluation over indexed XML documents

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Tree automata

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- Staircase join (used in MonetDB)
- Twig joins
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Tree automata

- Streaming XML queries
- XML typechecking
- XML pattern-matching
- Theoretical work (expressivity or complexity results)

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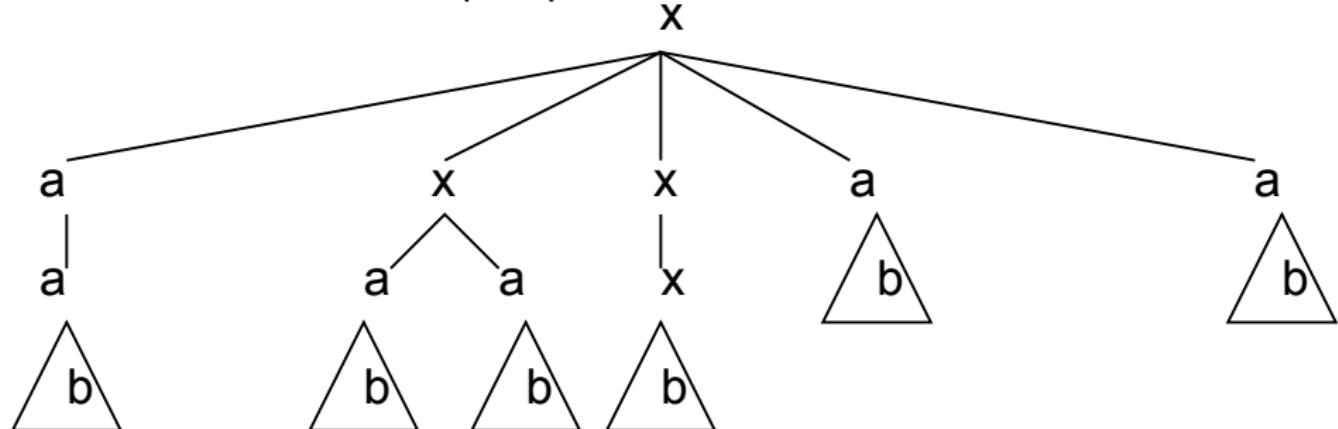
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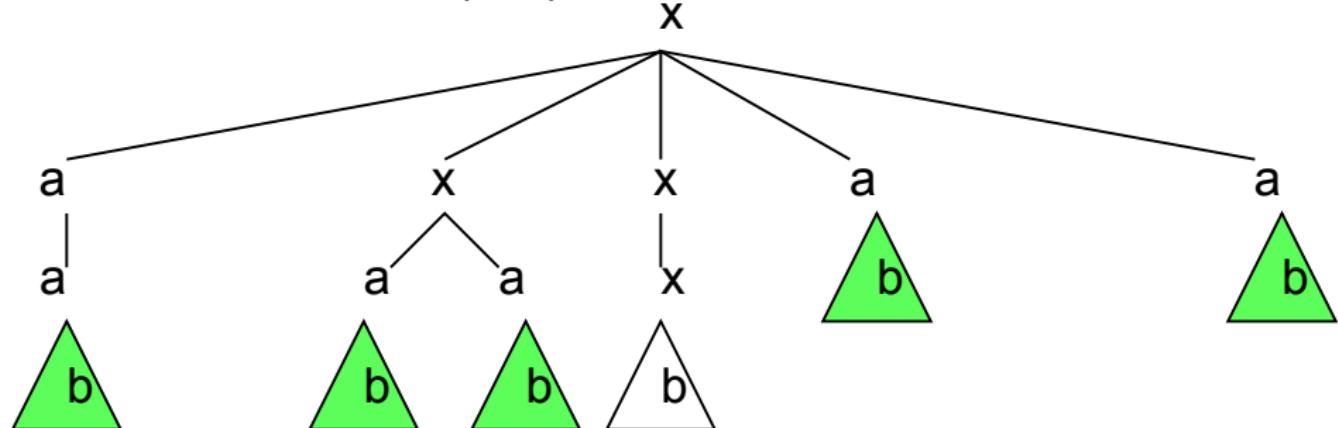
$|D|$  : do we need to visit the whole document ?

$|Q|$  : for each visited node, how much work is needed ?

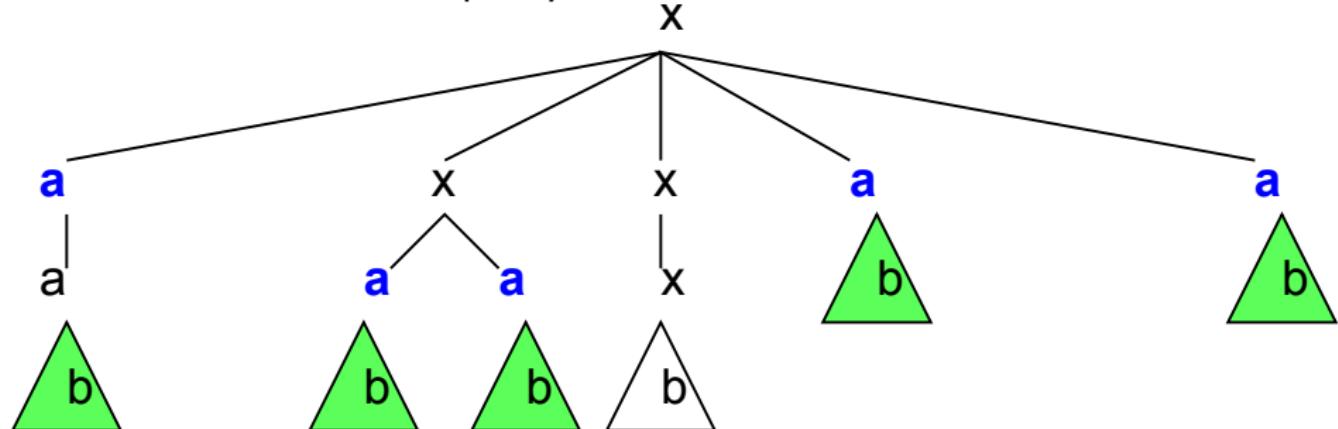
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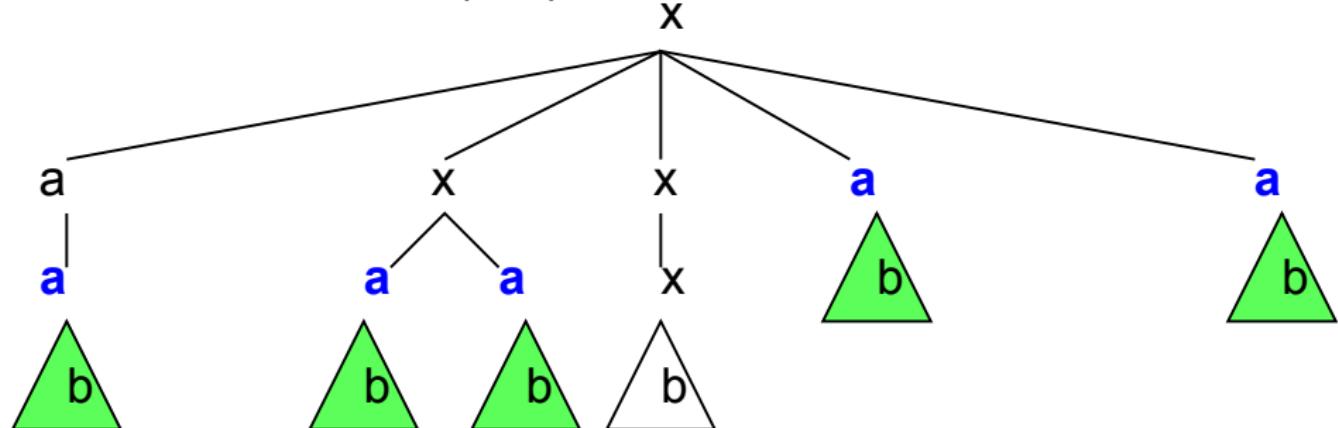
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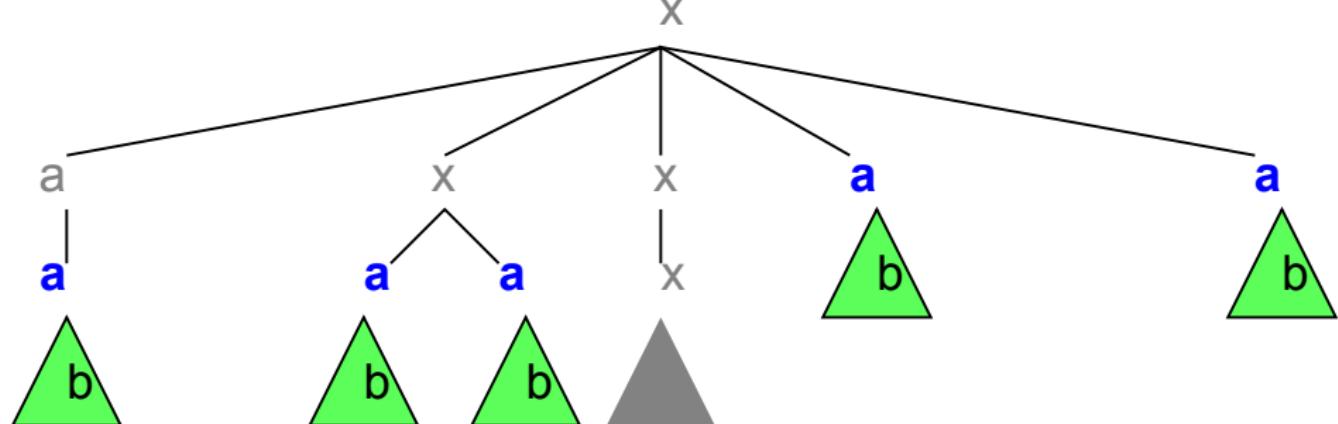
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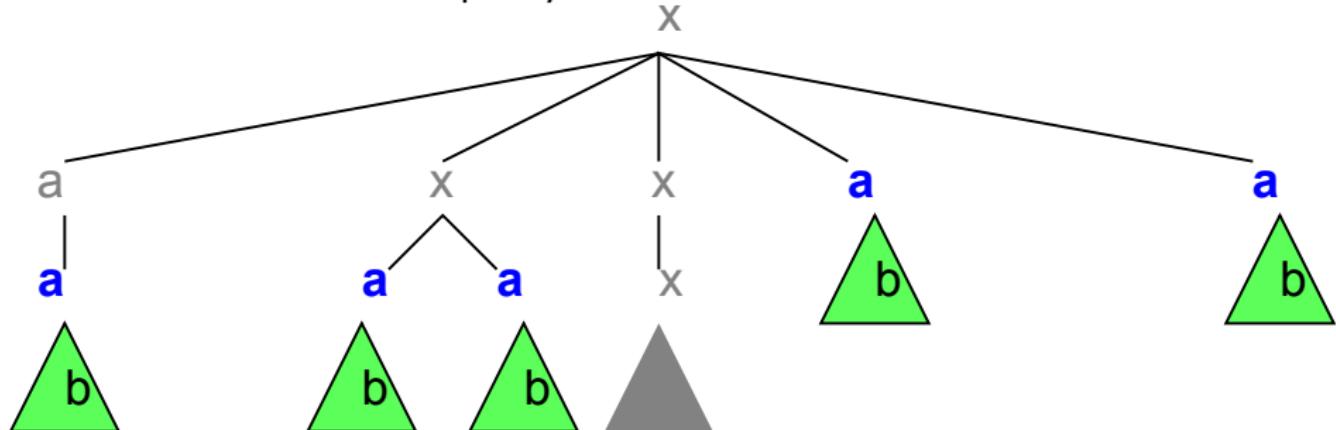
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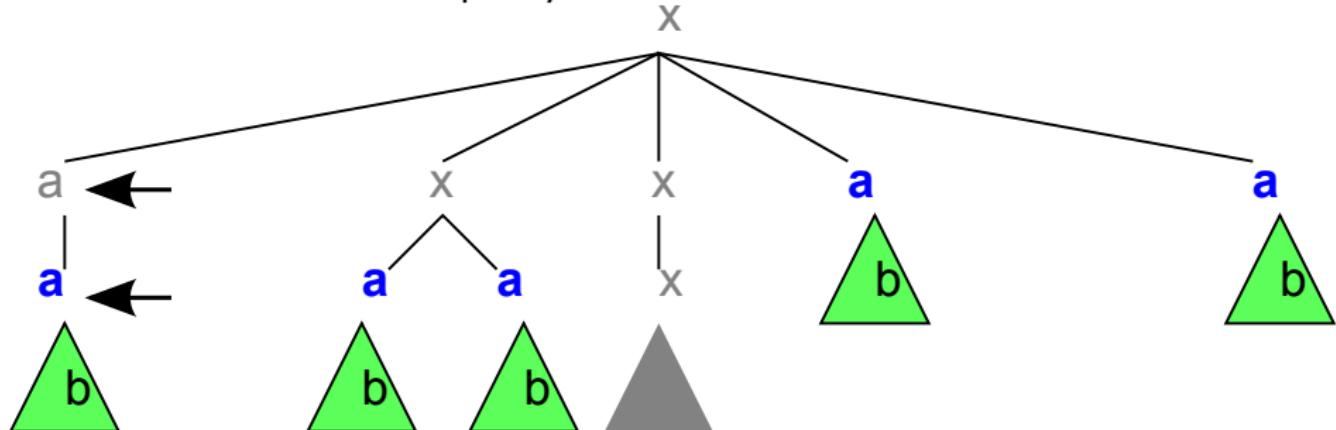


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- Does the engine need to know about **these nodes** ?
- How does it choose between to equally good nodes ?

- 1 Introduction**
- 2 Notations and definitions**
- 3 Relevant nodes**
  - Characterisation
  - Top-down deterministic relevance
  - Bottom-up deterministic relevance
- 4 XPath queries**
  - Top-down approximation
  - Implementation techniques
- 5 Experiments**
- 6 Conclusion**

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## Definition (Binary tree)

Binary trees over  $\Sigma$  :  $T(\Sigma)$  smallest set s.t. :

- $\# \in \Sigma$  (leaf symbol)
- $t_1, t_2 \in T(\Sigma) \Rightarrow \forall l \in \Sigma, l(t_1, t_2) \in T(\Sigma)$

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## Definition (Set of nodes of a tree $t$ )

Smallest set  $\text{Dom}(t) \subseteq \{1, 2\}^*$  such that :

- $\epsilon \in \text{Dom}(t)$
- $t(\epsilon) = l$  if  $t \equiv l(t_1, t_2)$
- $t(\epsilon) = \#$  if  $t \equiv \#$
- $t(i \cdot \pi) = t_i(\pi)$ , if  $t \equiv l(t_1, t_2)$

## Definition (Selecting Tree Automaton (STA))

$$\mathcal{A} = (\Sigma, Q, T, B, S, \delta)$$

- $\Sigma$  input symbols,  $Q$  set of states
- $T$  set of top states,  $B$  set of bottom-states
- $S \subseteq Q \times \Sigma$  set of selecting configurations
- $\delta$  set of transitions :  $q, L \rightarrow q_1, q_2$  (where  $L \subseteq \Sigma$ )

## Definition (Run $R$ of an STA $\mathcal{A}$ over $t$ )

A run is a mapping from  $\text{Dom}(t)$  to  $Q$  generated by  $\delta$

## Definition (Selected nodes)

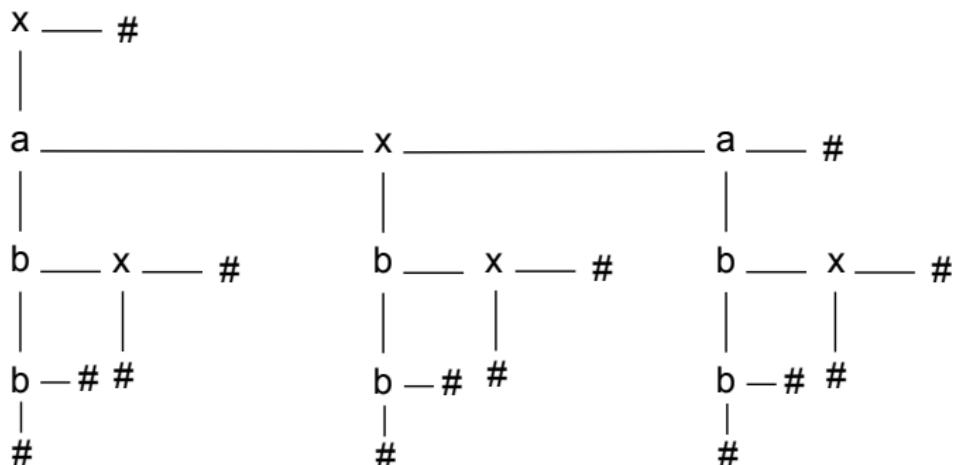
For a given run  $R$ , the set of selected nodes is

$$\mathcal{A}^R(t) = \{\pi \in \text{Dom}(t) \mid (R(\pi), t(\pi)) \in S\}$$

Example ( $//a//b$ )

$$\mathcal{A}_{//a//b} = (\Sigma, Q, T, B, S, \delta)$$

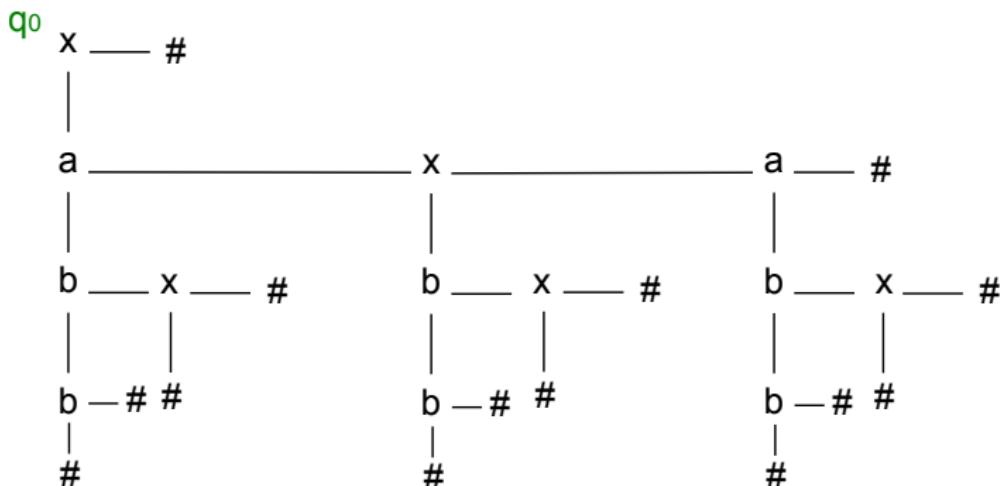
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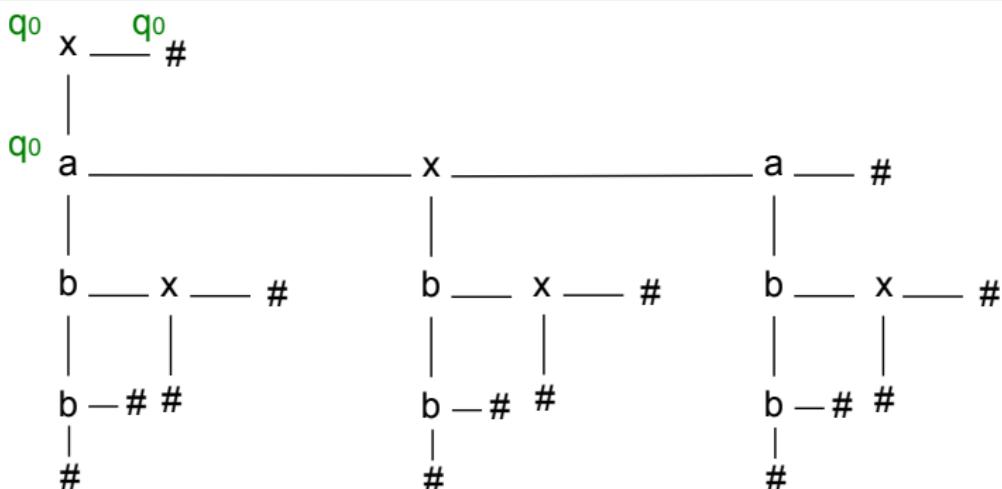
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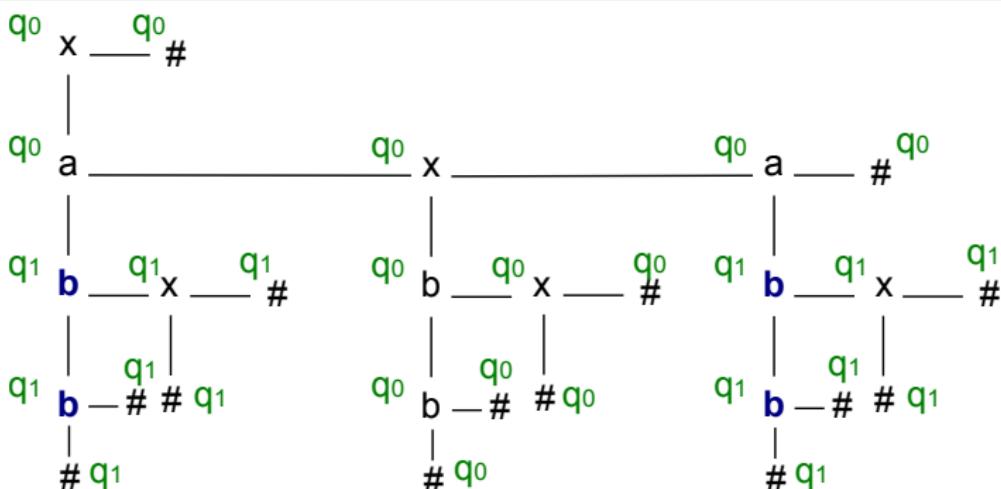
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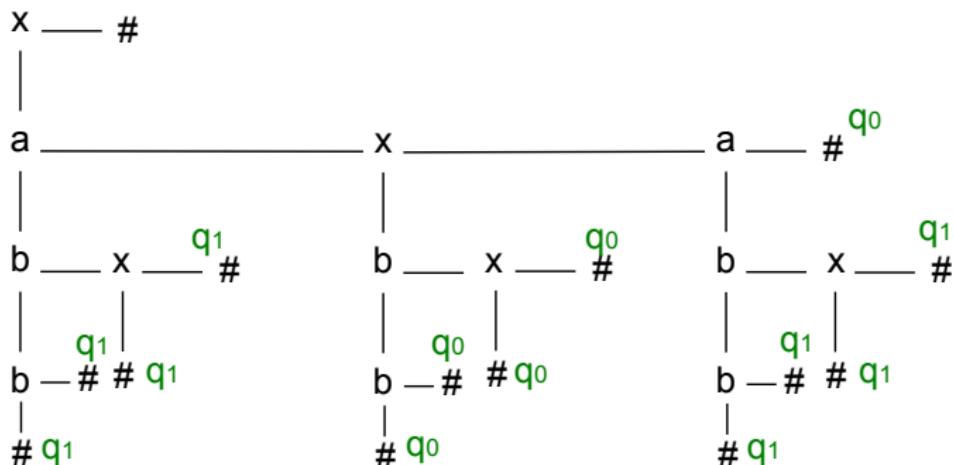
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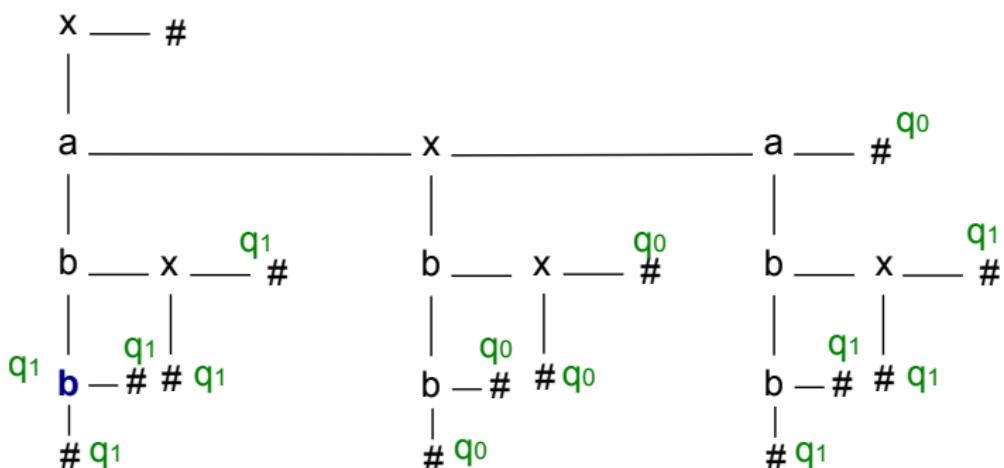
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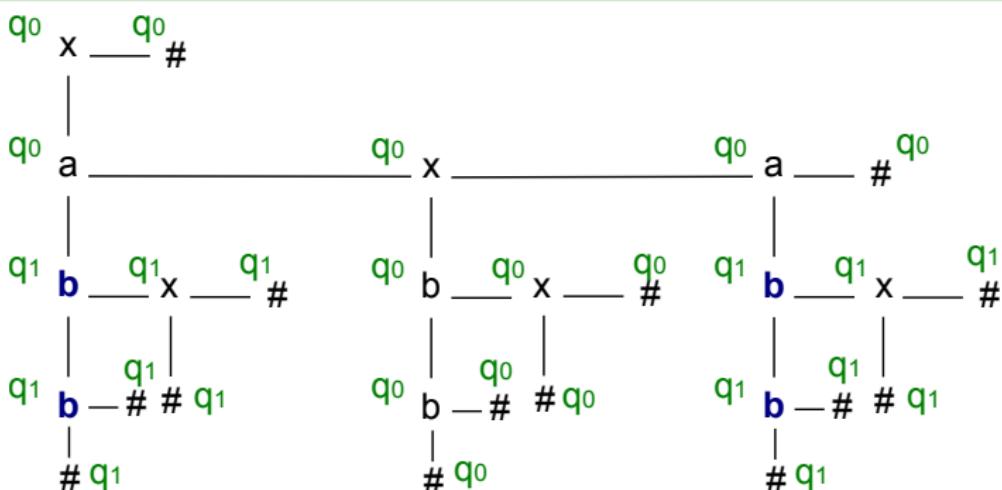


Example ( $//a//b$ )

$$\mathcal{A}_{//a//b} = (\Sigma, Q, \tau, \delta, S)$$

$\Sigma = \{a, b, x\}$        $Q = \{q_0, q_1\}$        $\tau = \{q_0\}$        $\delta = \{(q_0, q_1)\}$        $S = \{q_1\}$

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We write  $\mathcal{A}_T$  the universal recognizer (no selection)

## Definition (Equivalence)

$\mathcal{A} \equiv \mathcal{A}'$  if and only if :

- $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$
- $\forall t \in T(\Sigma), \mathcal{A}(t) = \mathcal{A}'(t)$

## Definition (Restriction to a state)

Let  $\mathcal{A} = (\Sigma, Q, T, B, S, \delta)$ .  $\mathcal{A}[q] \equiv (\Sigma, Q, \{q\}, B, S, \delta)$ .

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### Definition (Relevant node)

- let  $\mathcal{A}$  be an STA,  $t \in T(\Sigma)$  and  $R$  an accepting run
- let  $\pi \in \text{Dom}(t)$  s.t  $\pi \cdot i \in \text{Dom}(t), i \in \{1, 2\}$

$\pi$  is relevant in  $R$  iff it is selected or none of the following hold :

- $\mathcal{A}[R(\pi)] \equiv \mathcal{A}[R(\pi \cdot 1)] \equiv \mathcal{A}[R(\pi \cdot 2)]$
- $\mathcal{A}[R(\pi)] \equiv \mathcal{A}[R(\pi \cdot 1)]$  and  $\mathcal{A}[R(\pi \cdot 2)] \equiv \mathcal{A}_T$
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So we can tell which nodes are relevant to a query...

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:-)

Checking STA equivalence is hard (EXPTIME-complete)

Non-det.  $\Rightarrow$  many runs  $\Rightarrow$  different sets of relevant nodes

Consider a minimal top-down deterministic STA (TDSTA)

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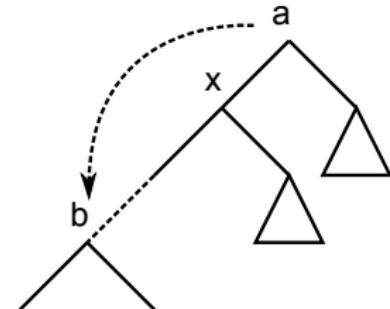
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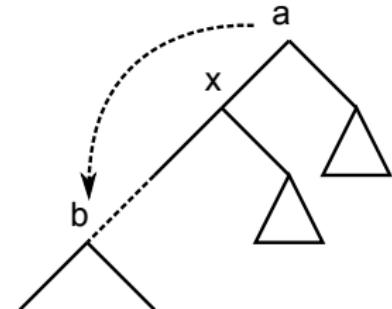
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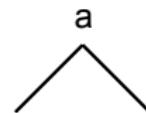
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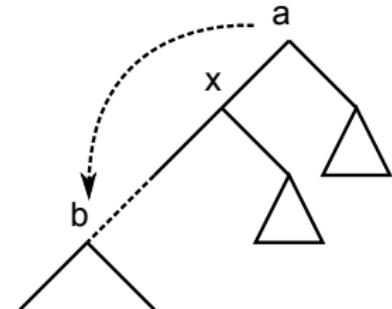
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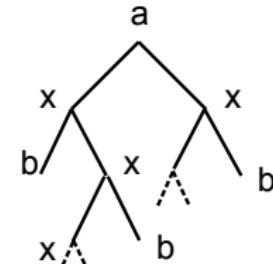
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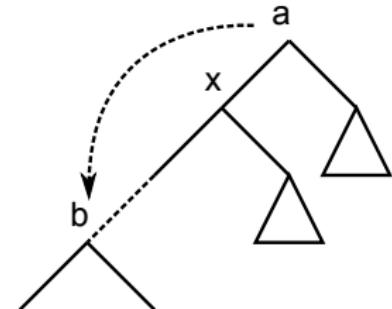
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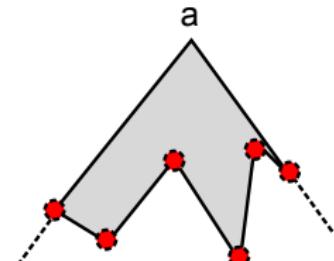
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- $//a//b$  is TD but not BD
- $//a[.//b]$  is BD but not TD

- Defined symmetrically by a state-change in the bottom-up run for a minimal BDSTA
- Automata moves :
  - bottom-most nodes with some label
  - lowest common ancestor of two nodes with some label
  - lowest node in the  $\uparrow_1 *$  or  $\uparrow_2 *$  direction with some label

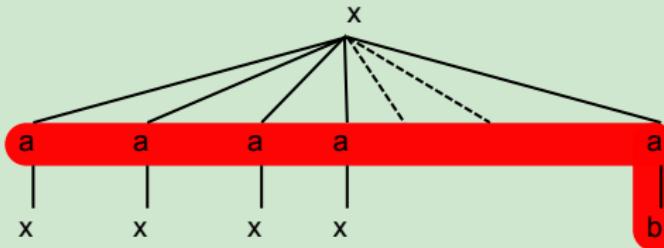
**Problem :** Top-down det. and bottom-up det. are incomparable and weaker than non-det.

## Example

- //a//b is TD but not BD
- //a[././/b] is BD but not TD
- //a//b[././/c] is neither TD nor BD

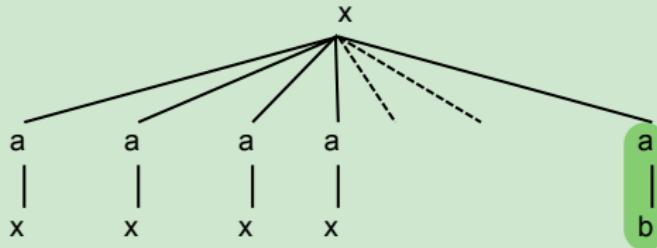
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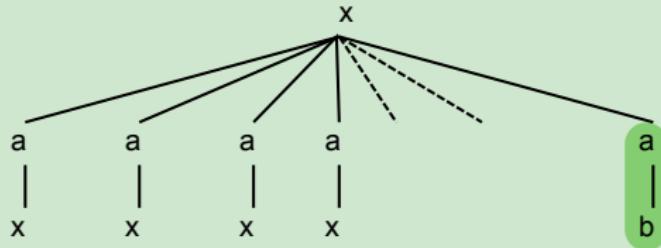
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$//a//b$  is **top-down** deterministic. However, there is a **bottom-up** non-deterministic run with less relevant nodes



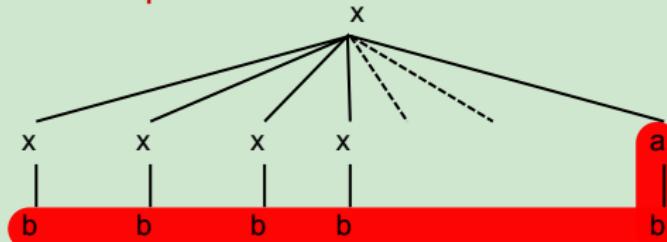
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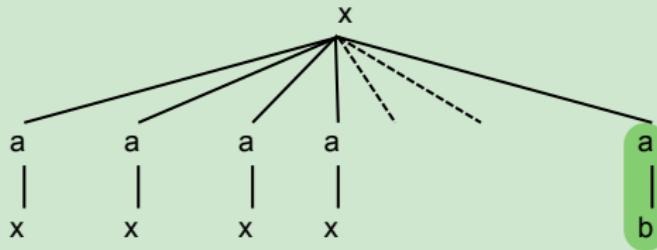
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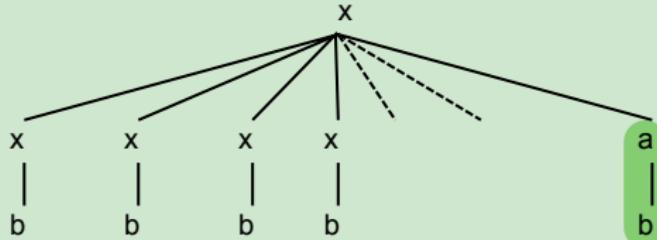


**Example  $(//a//b)$** 

$//a//b$  is **top-down** deterministic. However, there is a **bottom-up** non-deterministic run with less relevant nodes

**Example  $(//a[./.]/b)$** 

$//a[./.]/b$  is **bottom-up** deterministic. However, there is a **top-down** non-deterministic run with less relevant nodes



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## 2 Notations and definitions

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- Top-down deterministic relevance
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## 4 XPath queries

- Top-down approximation
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- transitions of the form  $q, L, \tau, \phi$
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## Example $(//a//b[c])$

$$\mathcal{A}_{//a//b(c)} = (\Sigma, \{q_0, q_1, q_2\}, \{q_0\}, \delta)$$

where  $\delta$  is :

$$\begin{array}{c|c|c} q_0, \{a\} \rightarrow \downarrow_1 q_1 & q_1, \{b\} \Rightarrow \downarrow_1 q_2 & q_2, \{c\} \rightarrow \top \\ q_0, \Sigma \rightarrow \downarrow_1 q_0 \vee \downarrow_2 q_0 & q_1, \Sigma \rightarrow \downarrow_1 q_1 \vee \downarrow_2 q_1 & q_2, \Sigma \rightarrow \downarrow_2 q_2 \end{array}$$

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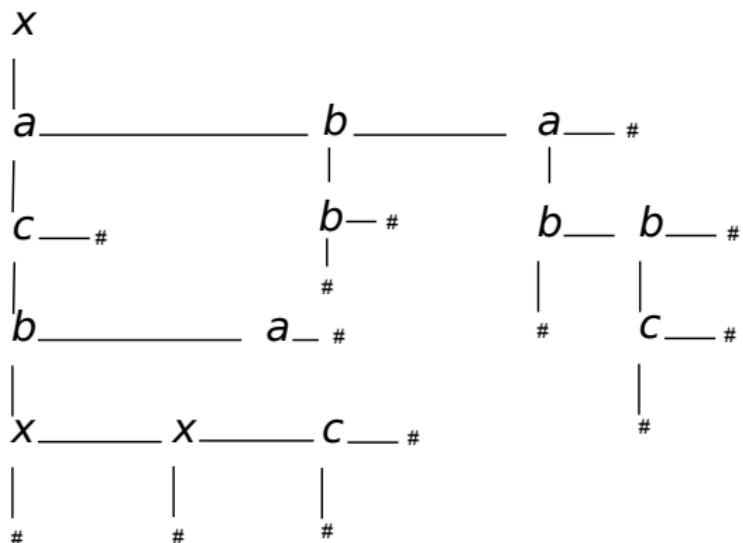
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We can use a textbook evaluation algorithm :  $O(|D| \cdot |\mathcal{A}|)$

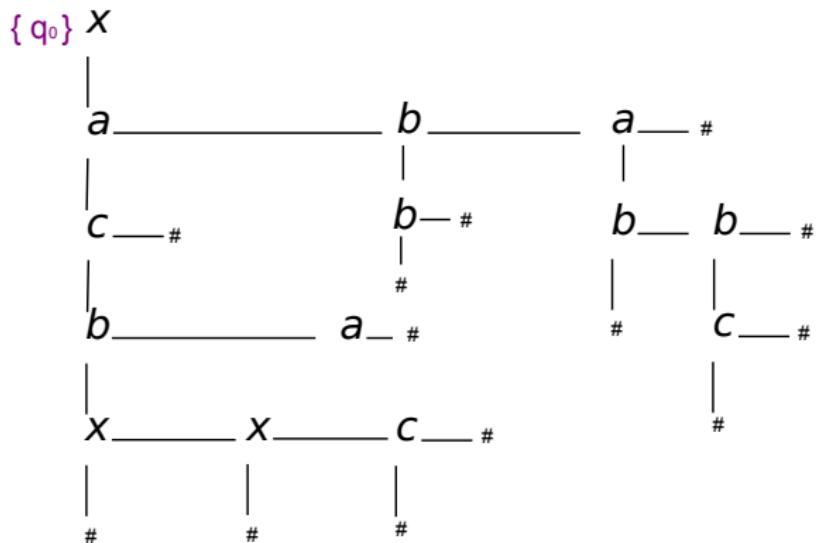
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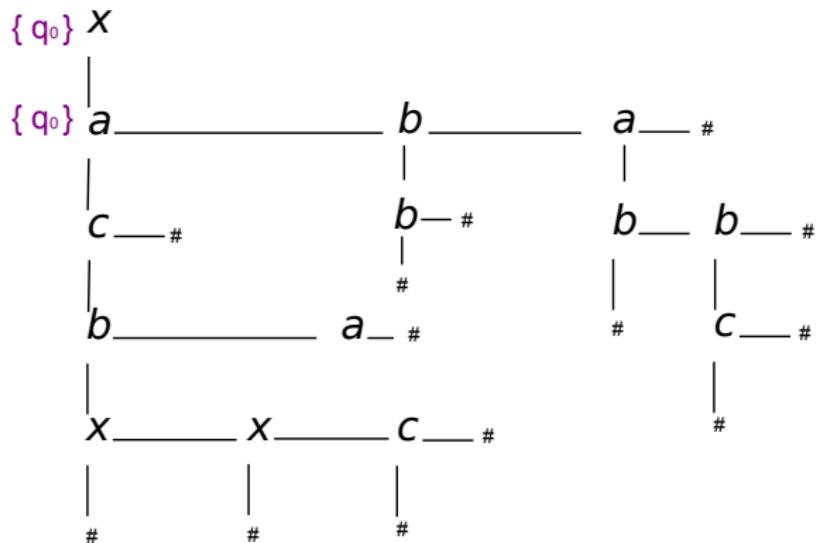
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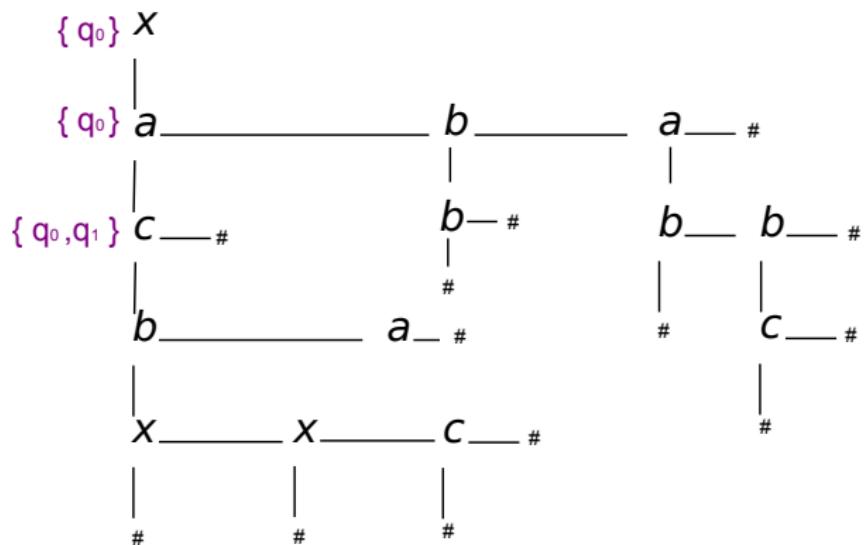
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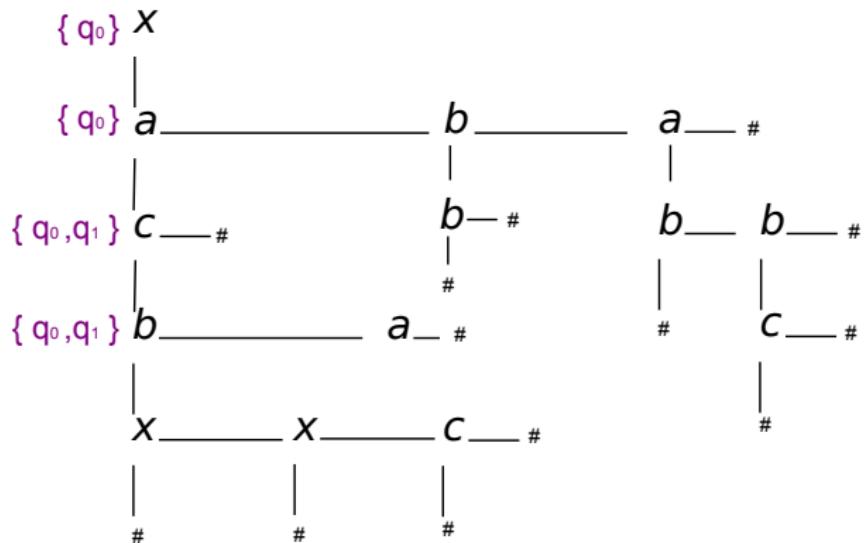
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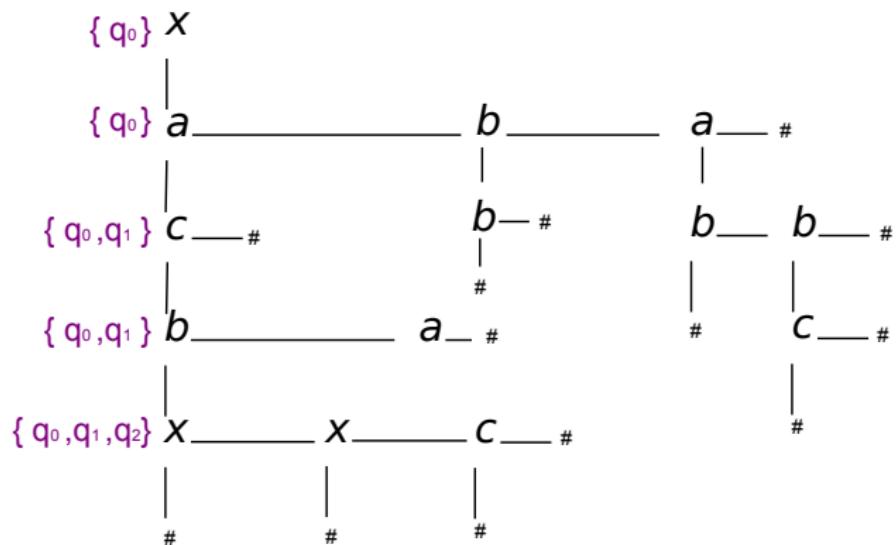
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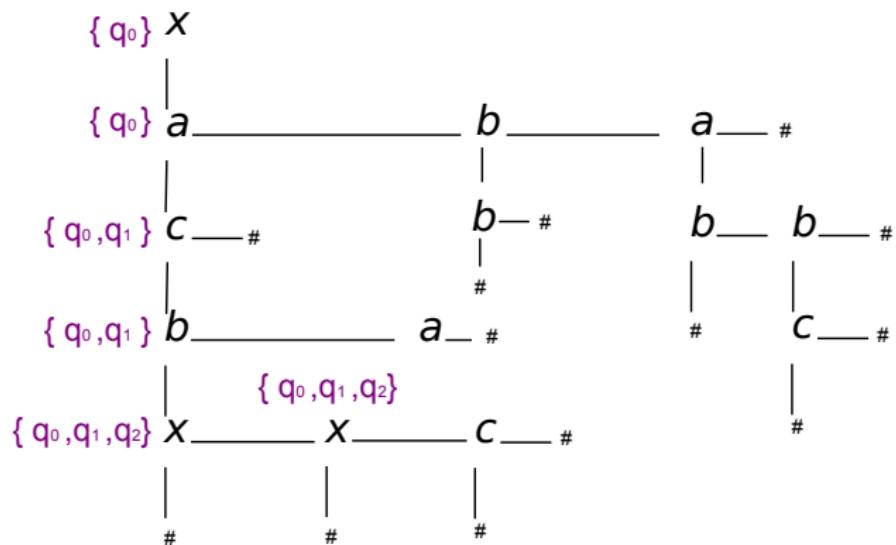
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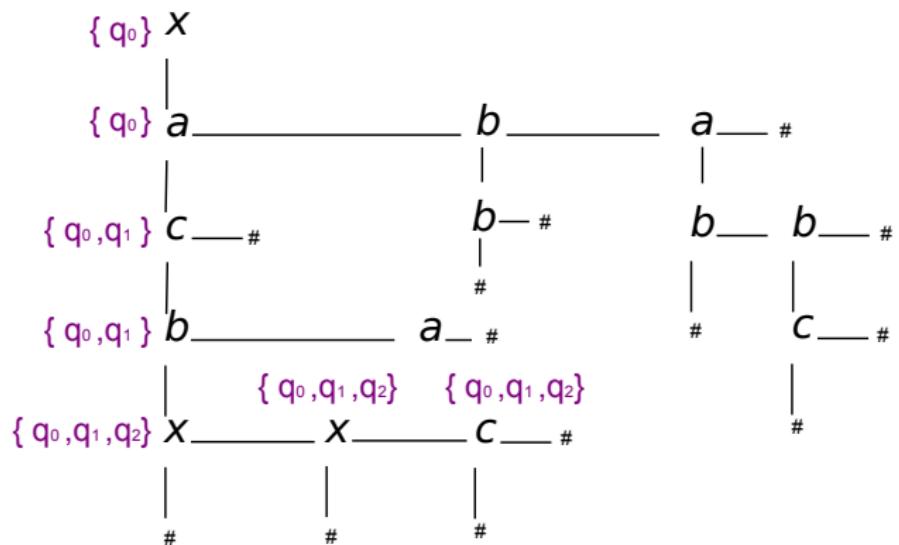
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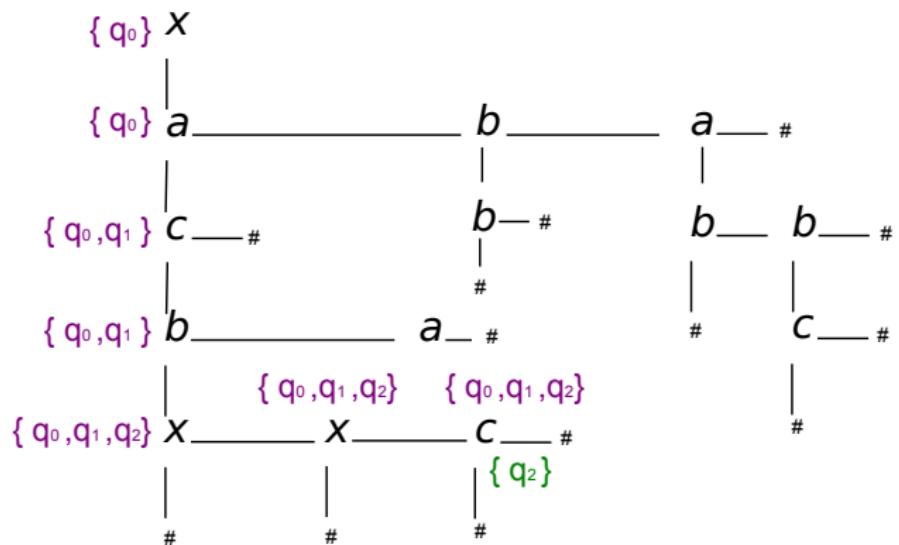
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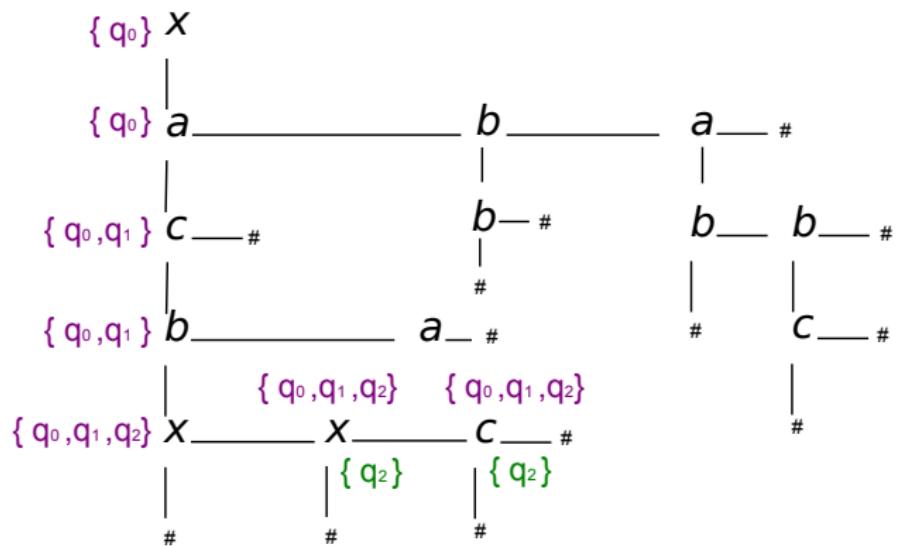
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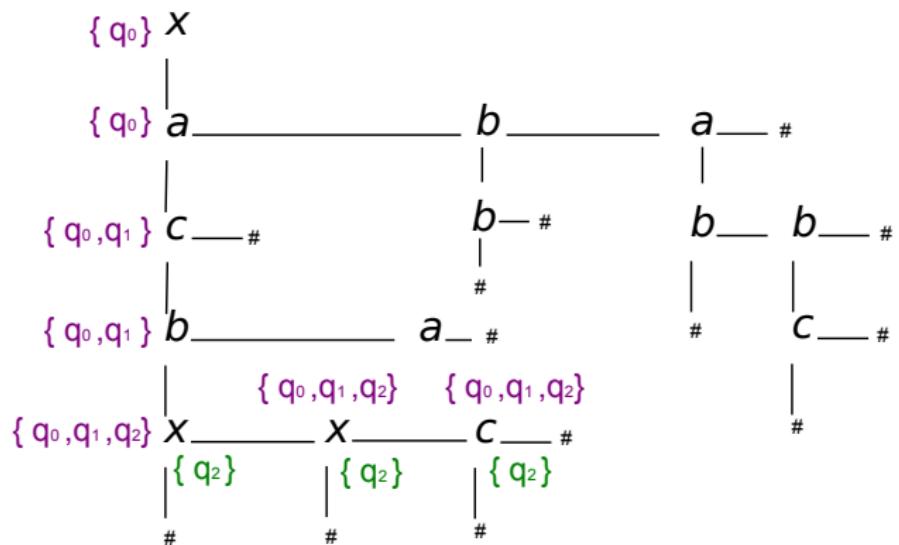
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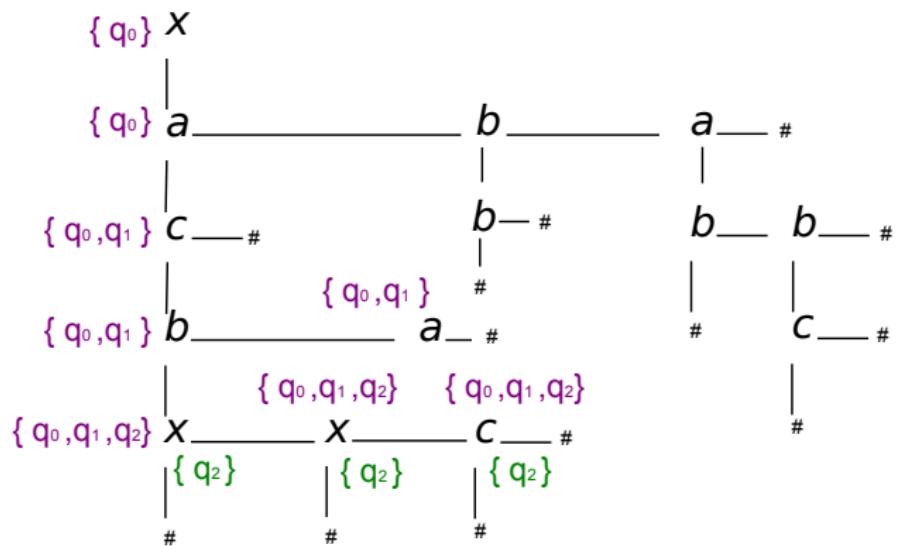
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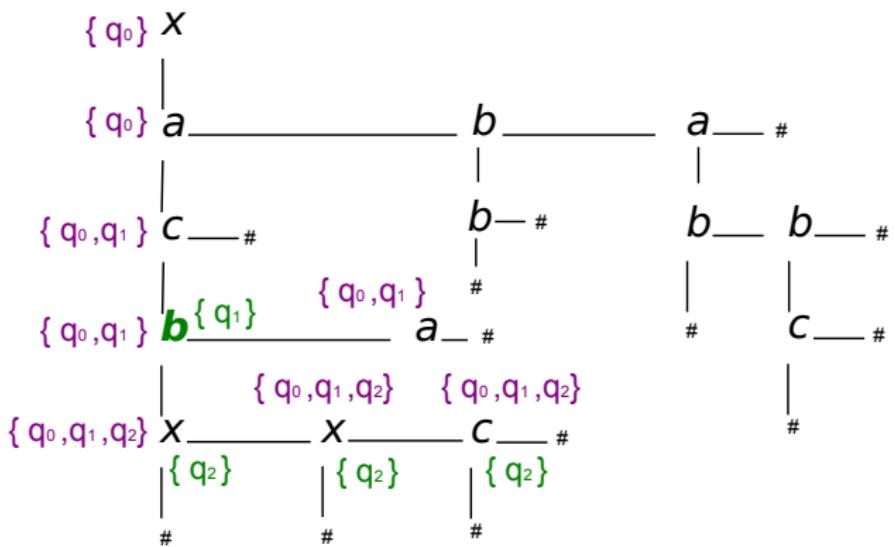
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 \end{array}
 \quad
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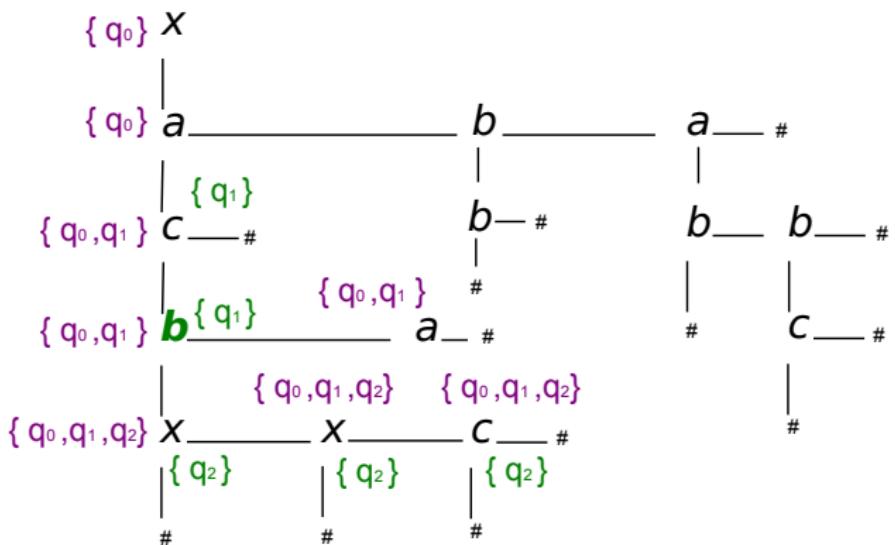
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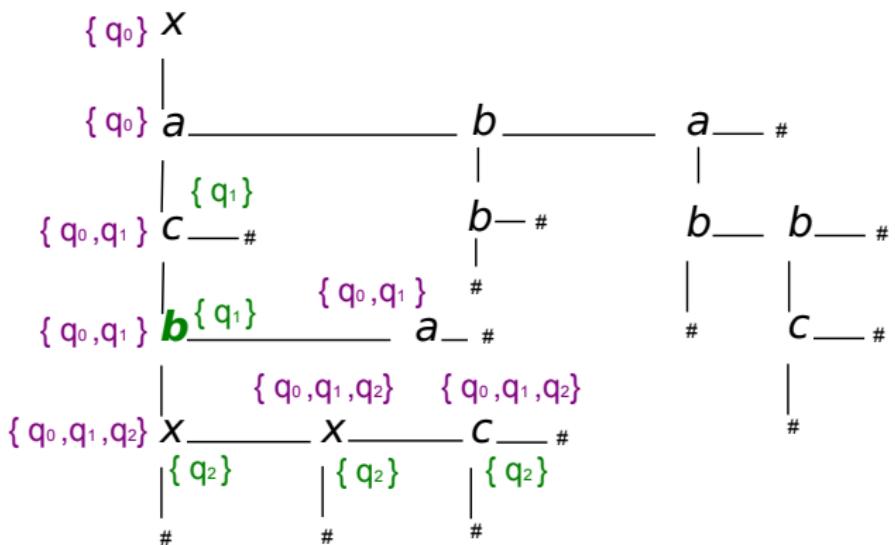
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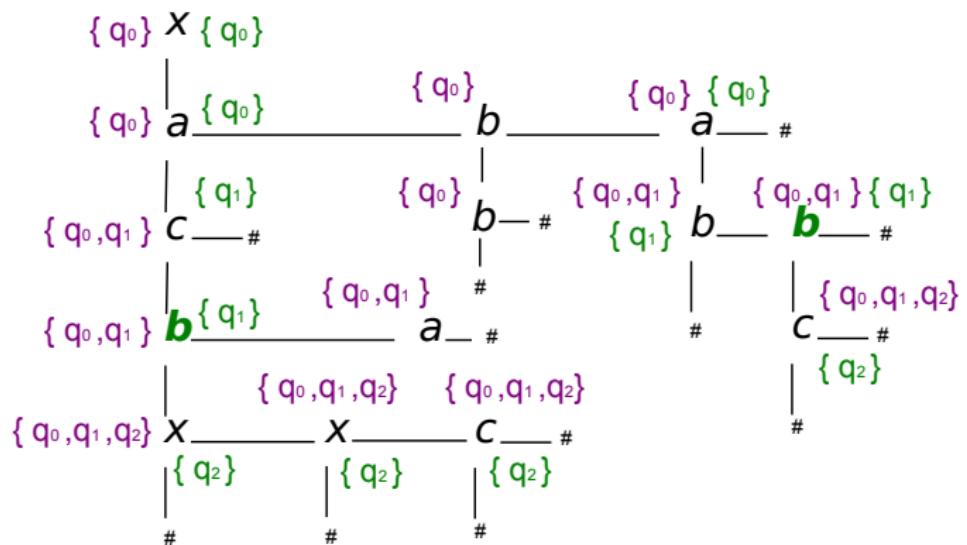
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 $\{q_0\} \times \{q_0\}$  $\{q_0\} \mid a \{q_0\}$  $\{q_0\}$  $b \_\_\_\_\_\_ \{q_0\}$  $a \_\_\_\# \{q_0\}$  $\{q_0, q_1\} \mid c \_\_\#\{q_1\}$  $\{q_0\}$  $b \_\_\#\{q_1\}$  $\{q_0, q_1\} \mid \{q_1\}$  $b \_\_\_ b \_\_\_ \{q_1\}$  $\{q_0, q_1\} \mid b \{q_1\}$  $\{q_0, q_1\}$  $a \_\_\#\{q_1\}$  $\#$  $\{q_0, q_1, q_2\} \mid X \{q_2\}$  $\{q_0, q_1, q_2\}$  $X \{q_2\}$  $C \_\_\#\{q_2\}$  $\# \{q_2\}$  $\# \{q_2\}$  $\# \{q_2\}$  $Q_0 = \{q_0\}$  $Q_1 = \{q_0, q_1\}$  $Q_2 = \{q_0, q_1, q_2\}$

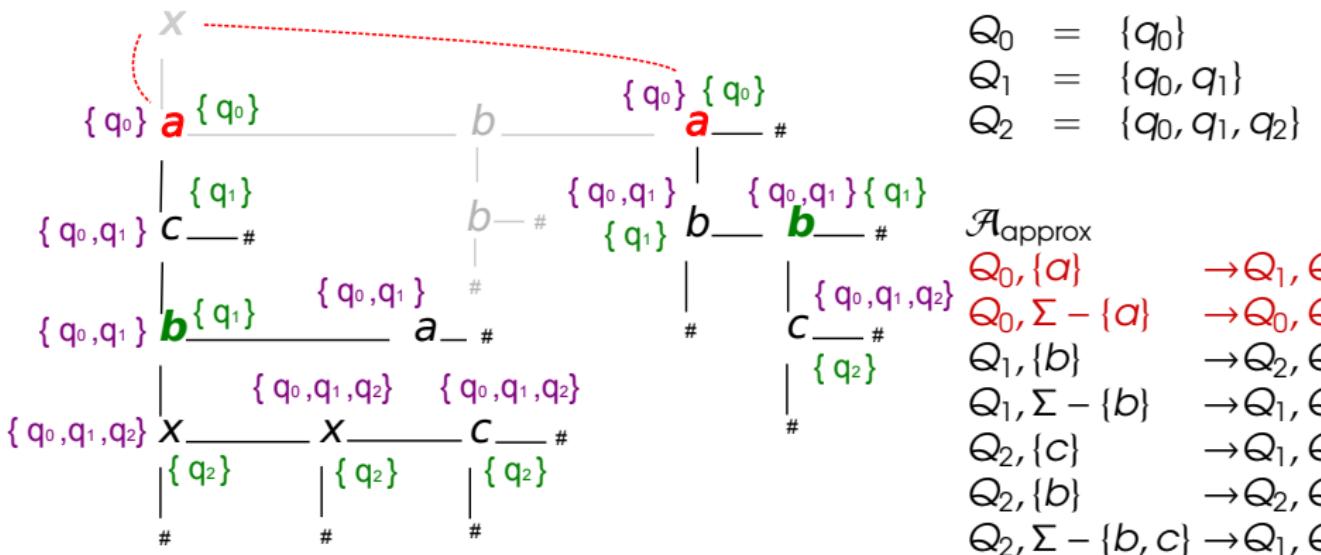
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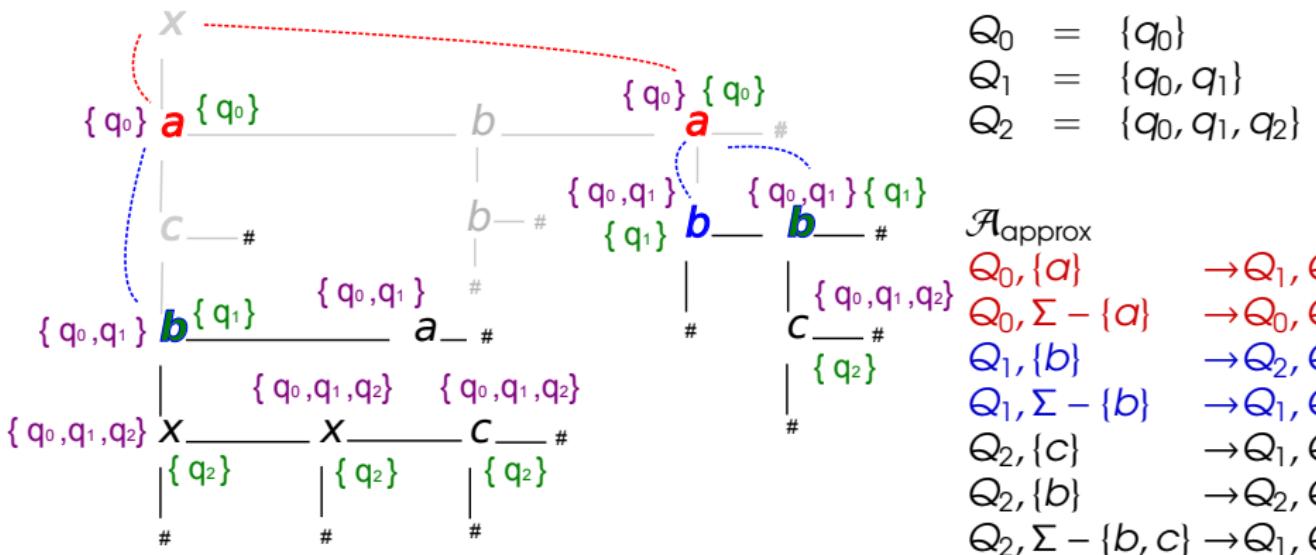
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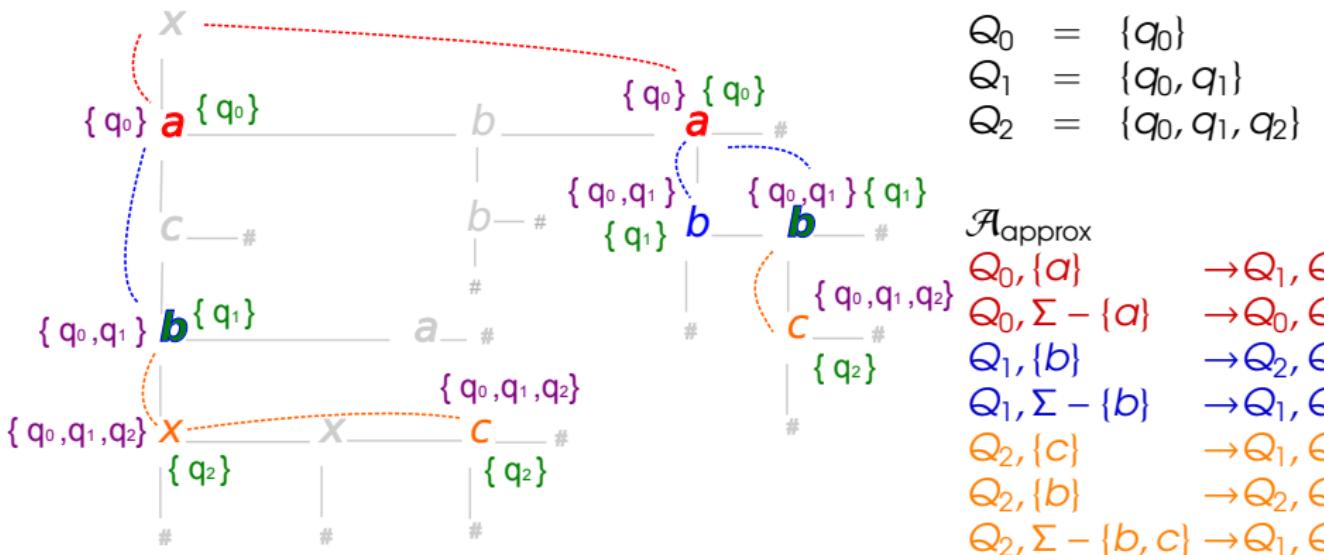
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Result-sets that allow  $O(1)$  duplicate-free sorted insert ( $|Q|$ )

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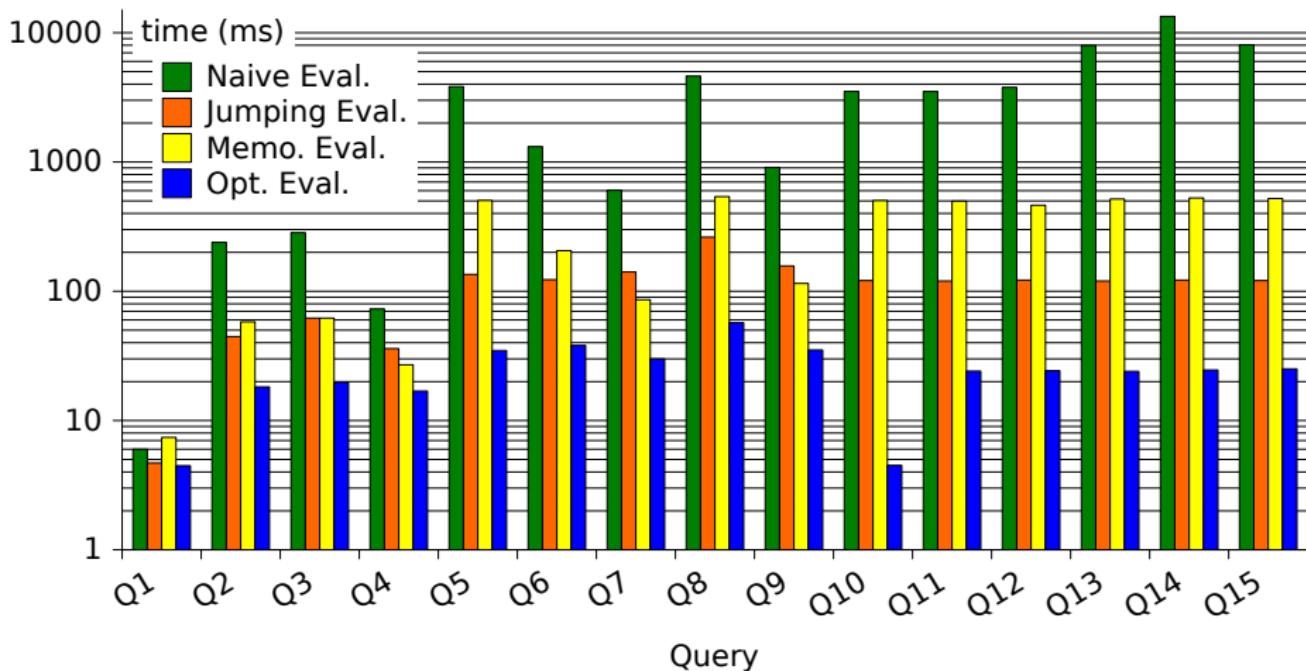
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Q1-Q10 taken from XPathMark

Q5-Q15 contain //

Factored computations :

Q10 :/site[ .//keyword ] touches two nodes, return 1 node

Q11 :/site//keyword touches 73071 nodes, returns 73070

Q12 :/site[.//keyword]//keyword touches 73071 nodes

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Reliance to syntactic changes

//a[ .//b or .//c ]//d and //a[ .//c or .//b ]//d

executed the same way (same nodes are touched)

## 1 Introduction

## 2 Notations and definitions

## 3 Relevant nodes

- Characterisation
- Top-down deterministic relevance
- Bottom-up deterministic relevance

## 4 XPath queries

- Top-down approximation
- Implementation techniques

## 5 Experiments

## 6 Conclusion

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- Can we characterize the average complexity ?