Using SMT Solvers for Deductive Verification of C and Java Programs

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The ProVal Project — http://proval.lri.fr/

- foundations of **ProVal**: the Coq project
  - type theory: type $\simeq$ logic specification
  - Curry-Howard isomorphism: proof $\simeq$ program
  - **functional** programs only
- goals of **ProVal**:
  - to deal with **imperative programs** (C, Java)
  - to apply our methods to **industrial cases**

Brief History

- 1999: a first approach for programs with side effects in Coq
- 2000-2003: EU project Verificard (verification of Java Card applets with industrial partners GemPlus, Schlumberger)
- 2001-: stand-alone WHY tool, to use both automatic and interactive provers
- 2003-: KRAKATOA tool for JAVA programs
- 2004-: CADUCEUS tool for C programs
- 2007: The WHY platform

Outline

1. overview of the Why platform
2. SMT solvers and program verification
   - theories of interest for program verification
3. SMT-lib and SMT-comp

Overview of the Why Platform

- general **goal**: prove behavioral properties of **pointer programs**
- pointer program = program manipulating data structures with **in-place mutable fields**
- we currently focus on **C** and **Java** programs

What Kind of Properties

two kinds

- **safety**, that is
  - no null pointer dereference
  - no array access out of bounds (no buffer overflow)
  - no division by zero
  - no arithmetic overflow
  - termination
- **behavioral correctness**
  - the program does what it is expected to do

Principles

- specification as **annotations** at the source code level
  - Java: an extension of JML (Java Modeling Language)
  - C: our own language (mostly JML-inspired)
- generation of **verification conditions** (VCs)
  - using Hoare logic / weakest preconditions
  - similar approaches: static ESC/Java, SPEC#, B method, etc.
- **multi-prover** approach
  - off-the-shelf provers, as many as possible
  - automatic provers (Alt-Ergo, Simplify, Yices, Z3, CVC3, etc.)
  - proof assistants (Coq, PVS, Isabelle/HOL, etc.)

A Toy Example: Binary Search

binary search: search a sorted array of integers for a given value

famous example; see J. Bentley's *Programming Pearls*
most programmers are wrong on their first attempt to write binary search

Binary Search (C code)

```
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1;
  while (l <= u ) {
    int m = (l + u) / 2;
    if (t[m] < v)
      l = m + 1;
    else if (t[m] > v)
      u = m - 1;
    else
      return m;
  }
  return -1;
}
```

Binary Search: Safety

- no division by zero
- no array access out of bounds
- termination

```
/*@ requires n >= 0 && \valid_range(t,0,n-1) */
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1;
  /*@ invariant 0 <= l && u <= n-1
    @ variant   u-l
    @*/
  while (l <= u ) {
    ...
}
```

DEMO

Binary Search: Behavioral Specification

```
/*@ requires
  @   n >= 0 && \valid_range(t,0,n-1) &&
  @   \forall int k1, int k2;
  @     0 <= k1 <= k2 <= n-1 => t[k1] <= t[k2]
  @ ensures
  @   (\result >= 0 && t[\result] == v) ||
  @   (\result == -1 &&
  @      \forall int k; 0 <= k < n => t[k] != v)
  @*/
int binary_search(int* t, int n, int v) {
  ...
}
```

Binary Search: Behavioral Specification (cont'd)

requires a stronger invariant

```
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1;
  /*@ invariant
    @   0 <= l && u <= n-1 &&
    @   \forall int k;
    @      0 <= k < n => t[k] == v => l <= k <= u
    @ variant u-l
    @*/
  while (l <= u ) {
    ...
  }
}
```

DEMO

Binary Search: Arithmetic Overflows

finally, let's prove that there is no arithmetic overflow... **there is one!**

in statement

```
int m = (l + u) / 2;
```

a possible overflow is signaled; a possible fix is

```
int m = l + (u - l) / 2;
```

see

- Google: "Read All About It: Nearly All Binary Searches and Mergesorts are Broken"
- "Types, Bytes, and Separation Logic" POPL'07

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Verification Technique

we use a standard technology (component-as-array memory model, weakest preconditions, etc.)

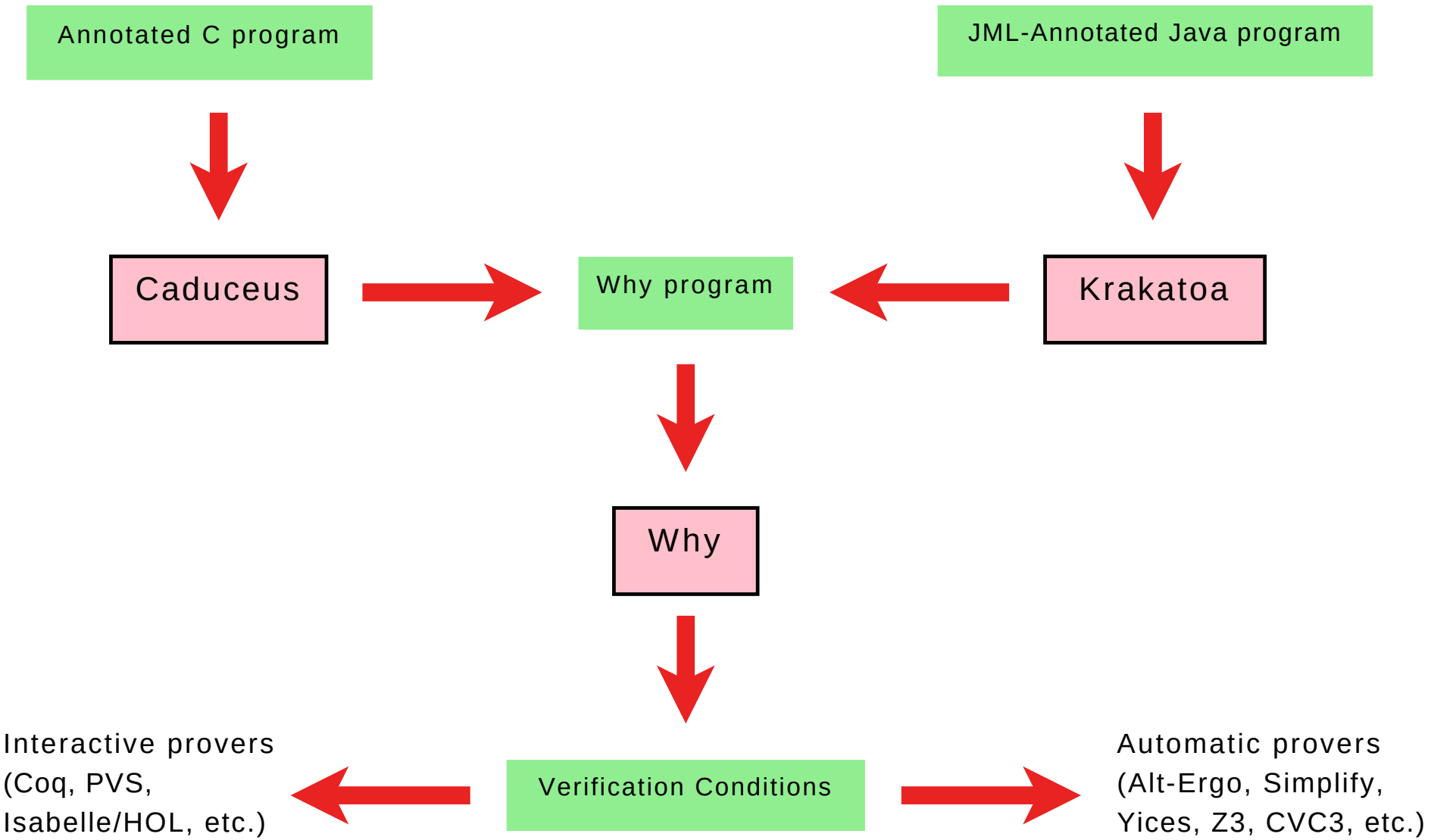
two specific issues:

- how to share the effort which is common to C and Java
- how to use many different theorem provers

our solution: the use of an intermediate language, **Why**, which is

- a VC generator
- a common front-end to various provers

Platform Overview



Why: a Verification Condition Generator

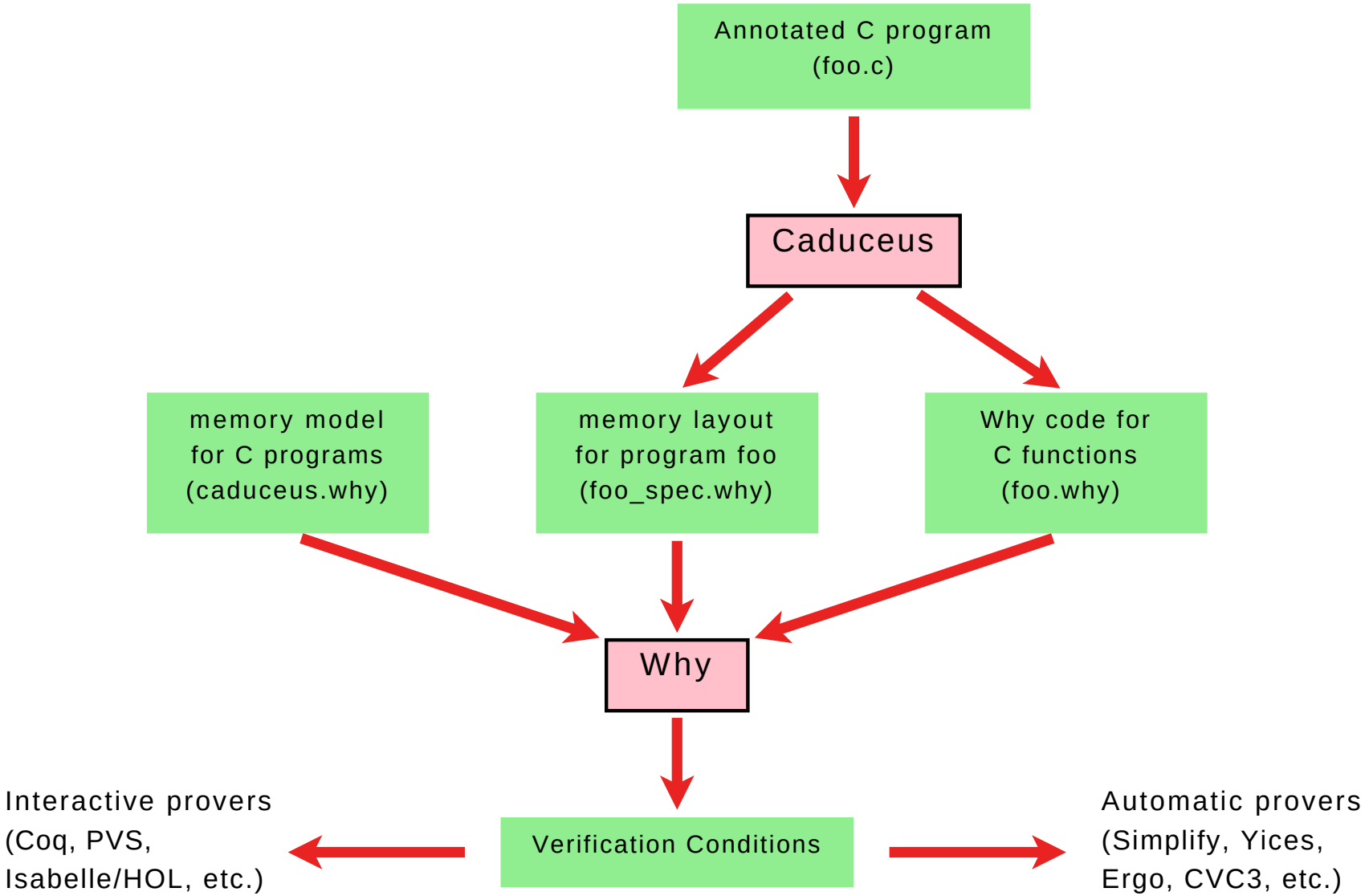
Why is a **verification condition generator** for a language with

- variables containing pure values, no alias (~ Hoare-logic language)
- usual control structures (loops, tests, etc.)
- exceptions
- (possibly recursive) functions
- polymorphic first-order logic with equality and arithmetic

Why is similar to Boogie (SPEC# project)

Why is also responsible for **translating** verification conditions to the **native logics** of all provers

Generating the Verification Conditions



SMT Solvers and Program Verification

Disclaimer

don't be mistaken by the remaining of this talk;

I **do** think that SMT solvers are great tools!

Which Logics for Program Verification

SMT solvers provide

- **first-order logic with equality**
  - memory models, user axiomatic models, etc.
- **integer/rational/real linear arithmetic**
  - integer arithmetic: array indices, pointer arithmetic, etc.
- **applicative arrays**
  - axiomatic approach is equally efficient
  - extensionality is not needed in practice
- **fixed-size bit vectors**
  - a too restrictive interface
- **tuples**, **records**, **inductive data types**

Which Logics for Program Verification

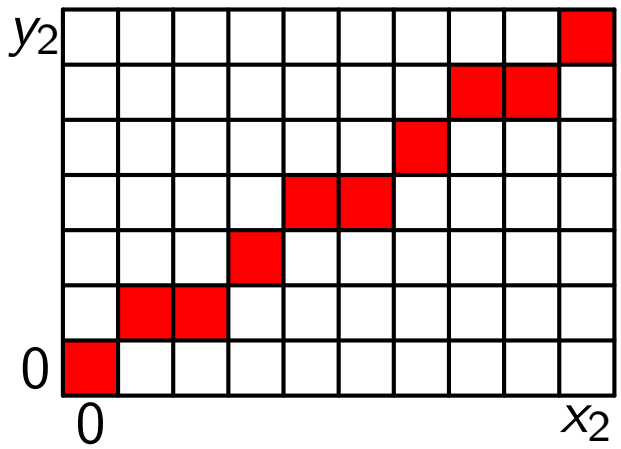
relevant theories for program verification can be different

- **non-linear arithmetic**
- **finite sets**
- **reachability**

let us consider some examples

- Bresenham's line drawing algorithm
- Dijkstra's shortest path algorithm

Example 1: Bresenham's Line Drawing Algorithm




```
logic x2,y2 : int
axiom first_octant : 0 <= y2 <= x2
```

x varies from 0 to x_2 ;
at each step, y is increased or not, according to the sign of e

```
parameter x,y,e : int ref
```

Example 1: Bresenham's Line Drawing Algorithm

```
let bresenham () =
  x := 0;
  y := 0;
  e := 2 * y2 - x2;
  while !x <= x2 do
    { invariant 0 <= x <= x2 + 1 and
        e = 2 * (x + 1) * y2 - (2 * y + 1) * x2 and
        2 * (y2 - x2) <= e <= 2 * y2 }
    (* here we would plot (x,y) *)
    if !e < 0 then
      e := !e + 2 * y2
    else begin
      y := !y + 1;
      e := !e + 2 * (y2 - x2)
    end;
    x := !x + 1
  done
```

Example 1: Bresenham's Line Drawing Algorithm

the code only uses linear arithmetic

the specification and thus the proofs require **non-linear arithmetic**

if suffices to add the following axioms

```
axiom z_ring_0 : forall a,b,c: int.  a * (b+c) = a*b + a*c
axiom z_ring_1 : forall a,b,c: int.  (b+c) * a = b*a + c*a
```

DEMO

Example 2: Dijkstra's Shortest Path

single-source shortest path in a weighted graph

$S \leftarrow \emptyset$
$Q \leftarrow \{src\}$;
$d[src] \leftarrow 0$
while $Q \backslash S$ not empty do
  extract u from $Q \backslash S$ with minimal distance $d[u]$
  $S \leftarrow S \cup \{u\}$
  for each vertex v such that $u \xrightarrow{w} v$
    $d[v] \leftarrow \min(d[v], d[u] + w)$
    $Q \leftarrow Q \cup \{v\}$

Example 2: Dijkstra's Shortest Path

finite sets are everywhere in the code/specification:

- set of vertices V
- set of successors of u
- sets S and Q

all we need is

- the empty set $\emptyset$
- addition $\{x\} \cup s$
- subtraction $s \backslash \{x\}$
- membership predicate $x \in s$

Example 2: Dijkstra's Shortest Path

```
type 'a set

logic set_empty : 'a set
logic set_add : 'a, 'a set -> 'a set
logic set_rmv : 'a, 'a set -> 'a set
logic In : 'a, 'a set -> prop

predicate Is_empty(s : 'a set) =
  forall x: 'a. not In(x, s)

predicate Incl(s1 : 'a set, s2 : 'a set) =
  forall x: 'a. In(x, s1) -> In(x, s2)
```

Example 2: Dijkstra's Shortest Path

```
axiom set_empty_def :
  Is_empty(set_empty)

axiom set_add_def :
  forall x: 'a.  forall y: 'a.  forall s: 'a set.
  In(x, set_add(y,s)) <-> (x = y or In(x, s))

axiom set_rmv_def :
  forall x: 'a.  forall y: 'a.  forall s: 'a set.
  In(x, set_rmv(y,s)) <-> (x <> y and In(x, s))
```

Example 2: Dijkstra's Shortest Path

termination requires the notion of cardinality

```
logic set_card : 'a set -> int

axiom card_nonneg : forall s: 'a set. set_card(s) >= 0

axiom card_set_add :
  forall x: 'a. forall s: 'a set.
  not In(x,s) -> set_card(set_add(x,s)) = 1 + set_card(s)

axiom card_set_rmv :
  forall x: 'a. forall s: 'a set.
  In(x,s) -> set_card(s) = 1 + set_card(set_rmv(x, s))

axiom card_Incl :
  forall s1,s2 : 'a set.
  Incl(s1,s2) -> set_card(s1) <= set_card(s2)
```

Example 2: Dijkstra's Shortest Path

```
while ... do
  { ... variant set_card(V) - set_card(S) }
  ...
  S := set_add u !S;
  ...
  while ... do
    { ... variant set_card(su) }
    ...
    su := set_rmv v !su
  done
done
```

Finite Sets

a theory of **finite sets** with constant $\emptyset$, operations $\{x\} \cup s$, $s \backslash \{x\}$, $\mathrm{card}(s)$ and predicate $x \in s$ would be extremely useful (even if incomplete)

Example 2: Dijkstra's Shortest Path

```
(* paths *)

logic path : vertex, vertex, int -> prop

axiom path_nil :
  forall x: vertex.  path(x,x,0)

axiom path_cons :
  forall x,y,z: vertex.  forall d: int.
  path(x,y,d) -> In(z,g_succ(y)) ->
  path(x,z,d+weight(y,z))

(* and shortest paths *)

predicate shortest_path(x: vertex, y: vertex, d: int) =
  path(x,y,d) and forall d': int.  path(x,y,d') -> d <= d'
```

Example 2: Dijkstra's Shortest Path

```
axiom path_inversion :
  forall src,v: vertex.  forall d: int.  path(src,v,d) ->
    (v = src and d = 0) or
    (exists v': vertex.
      path(src,v',d - weight(v',v)) and In(v,g_succ(v')))

(* lemmas requiring induction *)

axiom length_nonneg :
  forall x,y: vertex.  forall d: int.  path(x,y,d) -> d >= 0

...
```

Reachability

more generally, a theory of **reachability** is often used when specifying programs

this is simply the **reflexive transitive closure** of some relation (requires a some kind of higher-order to be generic)

variants:

- paths without repetition
- paths with the list of nodes ($path(x, y, l)$)
- closure of a function
- etc.

Conversely

sometimes, no need for built-in theories

examples

- arrays
- machine arithmetic (fixed-size integers)
- bitwise arithmetic (low-level bit tricks)

Machine Arithmetic

3 possible models for C integer arithmetic in Why

- exact arithmetic
- bounded arithmetic (VCs to show absence of overflow)
- modulo arithmetic (faithful to program execution)

Bounded Arithmetic

```
type int32

logic of_int32: int32 -> int

axiom int32_domain :
 forall x: int32.  -2147483648 <= of_int32(x) <= 2147483647

parameter int32_of_int :
  x: int ->
    { -2147483648 <= x <= 2147483647 }
    int32
    { of_int32(result) = x }
```

Bounded Arithmetic

a C operation such as $x + y$ is translated into

```
int32_of_int(of_int32(x) + of_int32(y))
```

which produces the verification condition

```
-2147483648 <= of_int32(x) + of_int32(y) <= 2147483647
```

no real need for a built-in theory

Modulo Arithmetic

```
type int32
logic of_int32: int32 -> int
axiom int32_domain : ...

logic mod_int32: int -> int

parameter int32_of_int :
  x: int -> { } int32 { of_int32(result) = mod_int32(x) }

axiom mod_int32_id :
  forall x: int.
  -2147483648 <= x <= 2147483647 -> mod_int32(x) = x

axiom mod_int32_def :
  forall x,k: int.
  mod_int32(x) = mod_int32(x + k * 4294967296)
```

Modulo Arithmetic

in some cases, you may only need

```
axiom mod_int32_gt :
  forall x: int.  x > 2147483647 ->
  mod_int32(x) = mod_int32(x - 4294967296)

axiom mod_int32_lt :
  forall x: int.  x < -2147483648 ->
  mod_int32(x) = mod_int32(x + 4294967296)
```

otherwise, you need

- either non-linear arithmetic
- or at a built-in theory of modulo arithmetic

Bitwise Arithmetic

challenge for **the verified program of the month**:

```
t(a,b,c){int d=0,e=a&~b&~c,f=1;if(a)for(f=0;d=(e-=d)&-e;f+=t(a-d,(b+d)*2,(
c+d)/2));return f;}main(q){scanf("%d",&q);printf("%d\n",t(~(~0<<q),0,0));}
```

Unobfuscating...

```
int t(int a, int b, int c) {
  int d, e=a&~b&~c, f=1;
  if (a)
    for (f=0; d=e&-e; e-=d)
      f += t(a-d, (b+d)*2, (c+d)/2);
  return f;
}

int main(int q) {
  scanf("%d", &q);
  printf("%d\n", t(~(~0<<q), 0, 0));
}
```

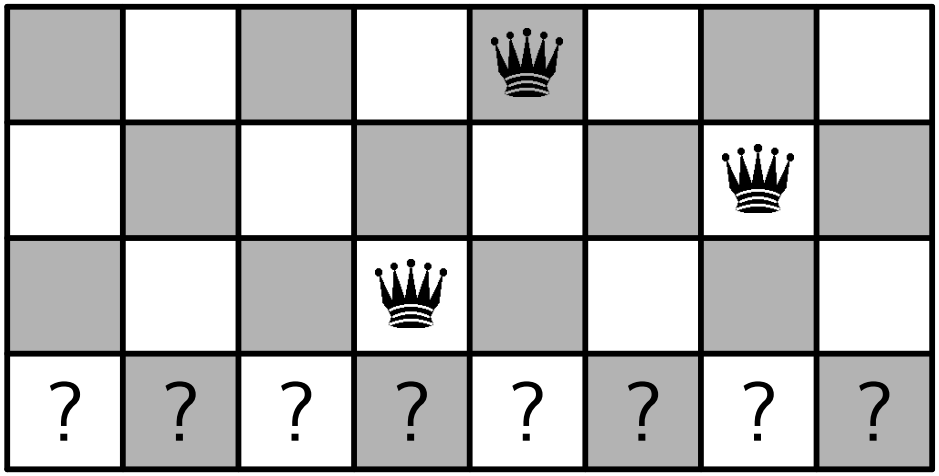
this program reads an integer n
and prints the number of solutions to the n-queens problem

How does it work?

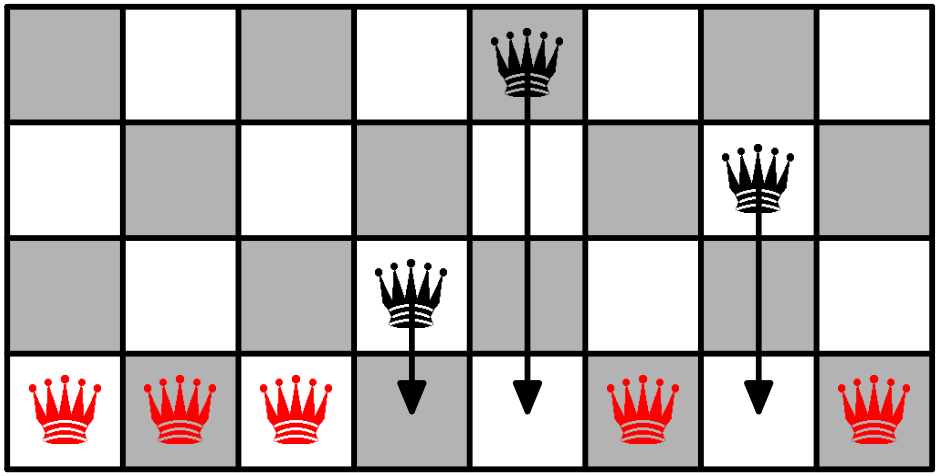
- backtracking algorithm (no better way to solve the n-queens)
- integers used as **sets** (bit vectors)

integers	sets
`0`	$\emptyset$
`a&b`	$a \cap b$
`a+b`	$a \cup b$, when $a \cap b = \emptyset$
`a-b`	$a \setminus b$, when $b \subseteq a$
`~a`	$\complement a$
`a&-a`	$min\_elt(a)$, when $a \neq \emptyset$
`~(~0<<n)`	$\{0, 1, \ldots, n-1\}$
`a*2`	$\{i+1 \mid i \in a\}$, written $S(a)$
`a/2`	$\{i-1 \mid i \in a \wedge i \neq 0\}$, written $P(a)$

What a, b and c mean

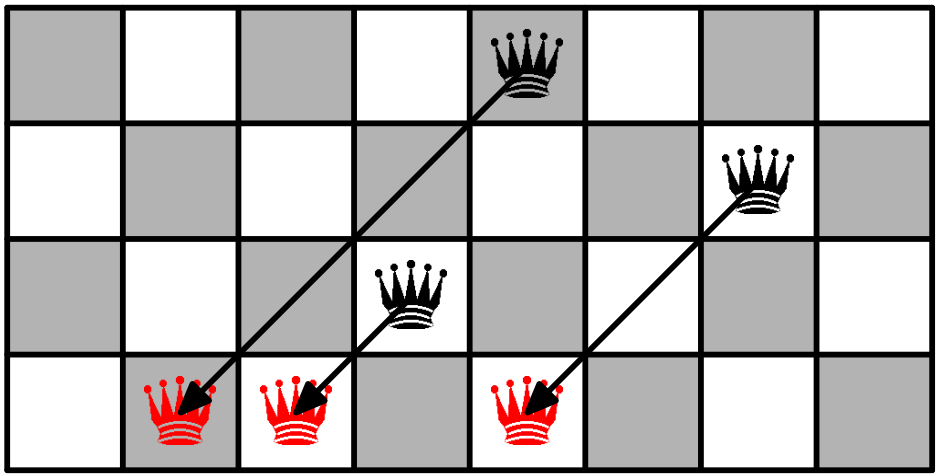


What a, b and c mean



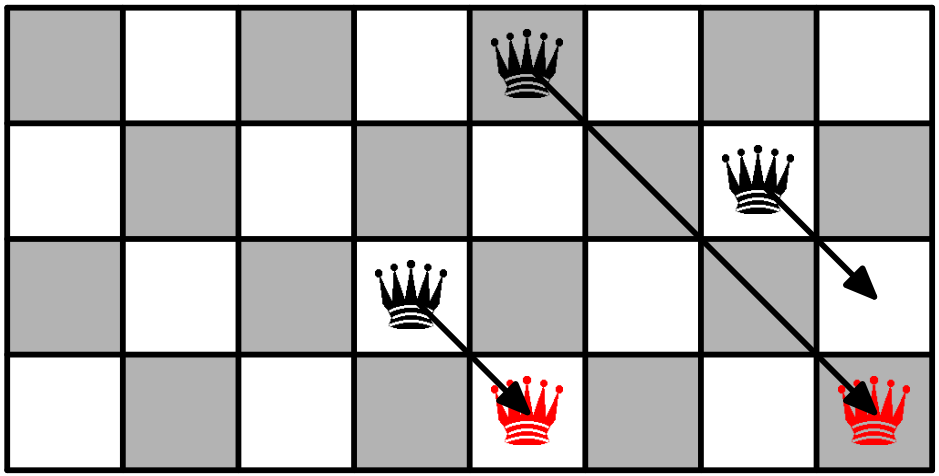
a = columns to be filled = 11100101_2

What a, b and c mean



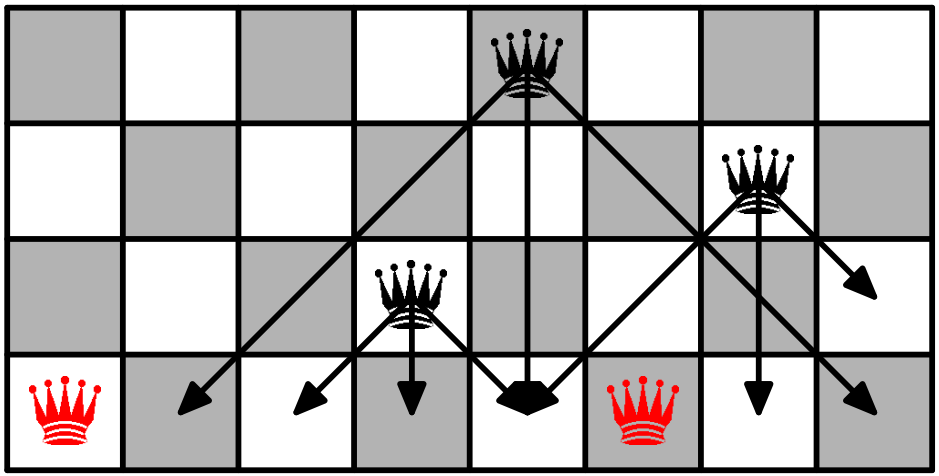
b = positions to avoid because of left diagonals = 01101000_2

What a, b and c mean



c = positions to avoid because of right diagonals = 00001001_2

What a, b and c mean



$a \,\&\tilde{}\, b \,\&\tilde{}\, c$ = positions to try = 10000100_2

Now it is clear

```
int t(int a, int b, int c) {
  int d, e=a&~b&~c, f=1;
  if (a)
    for (f=0; d=e&-e; e-=d)
      f += t(a-d, (b+d)*2, (c+d)/2);
  return f;
}

int queens(int n) {
  return t(~(~0<<n), 0, 0);
}
```

C ints as abstract sets

suppose we have axiomatized finite sets of integers (type `iset`)
now we interpret C ints as sets

```
//@ logic iset iset(int x)

/*@ axiom iset_c_zero :
  @   \forall int x; iset(x)==empty() <=> x==0 */

/*@ axiom iset_c_min_elt :
  @   \forall int x; x != 0 =>
  @   iset(x&-x) == singleton(min_elt(iset(x))) */

/*@ axiom iset_c_diff :
  @   \forall int a, int b;
  @   iset(a&~b) == diff(iset(a), iset(b)) */
...
```

Termination

```
int t(int a, int b, int c){
  int d, e=a&~b&~c, f=1;
  if (a)
    //@ variant card(iset(e-d))
    for (f=0; d=e&-e; e-=d) {
      f += t(a-d,(b+d)*2,(c+d)/2);
    }
  return f;
}
```

3 verification conditions, all proved automatically

similarly for the termination of the recursive function:
7 verification conditions, all proved automatically

Soundness

Safety, termination and correctness require **256** lines of code and specification

VCs:

- main function `queens`: **15** VCs
  - **all** proved automatically (Simplify, Alt-Ergo or Yices)
- recursive function `t`: **51** verification conditions
  - **42** proved automatically: 41 by Simplify, 37 by Alt-Ergo and 35 by Yices
  - **9** proved manually using Coq (and Simplify)

SMT-lib and SMT-comp

SMT-lib

SMT-lib is one possible output of Why, and is used for Yices, Z3 and CVC3

our common language is **Why**'s logic

- **polymorphic** first-order logic
- triggers
- predicate definitions
- datatype for booleans $\neq$ type of propositions
- nice syntax with infixes
- output for provers that do not support SMT-lib such as Simplify, Zenon, etc.

SMT-comp

some benchmarks are somewhat artificial and test SMT solvers in some situations that are not realistic
(of course, we are to be blamed first, for not providing more benchmarks in program verification)

if a SMT solver cannot win a SAT competition, it is ok for us

more emphasis should be put on

- categories with **quantifiers** (memory models, user axiomatic models)
- problems with lots of irrelevant hypotheses

conclusion

Conclusion

SMT solvers are great tools, and they changed our live

realistic program verification can now be done automatically

SMT solvers could be even more efficient

- built-in theories for program verification
  - finite sets
  - reachability
  - non-linear arithmetic
- selection of relevant hypotheses
- goal-directed proof search