# Pôle Preuves et langages

Jean-Christophe Filliâtre

19 octobre 2022

LMF
Laboratoire
Méthodes
Formelles

1. structure
2. scientific activities
3. focus (15 minutes) : A. Paskevich and J.-H. Jourdan
   Proof of programs with Why3 and Creusot
4. focus (15 minutes) : K. Nguyễn
   Occurence Typing and Set-theoretic types
   for dynamic languages

# members

- 26 permanent members


Idir

Thibaut

Bruno

Véronique

Frédéric

Valentin

Sylvie

Frédéric

Hubert

Sylvain

Évelyne

Gilles

Jean-Christophe

Armaël

Jacques-Henri

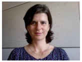
Chantal

Claude

Guillaume

Kim

Andrei

Christine

Mihaela

Safouan

Théo

Burkhart

Lina

- 4 postdocs and engineers
- 24 PhD students

- 4 teams
  - Arithmétique des ordinateurs (G. Melquiond)
  - Calcul, langages et compilation (K. Nguyễn)
  - Preuve de programmes (A. Paskevich)
  - Preuve mécanisée (C. Keller)
- 2 Inria EPC
  - Deducteam (G. Dowek)
  - Toccata (C. Marché)

Arithmétique des ordinateurs

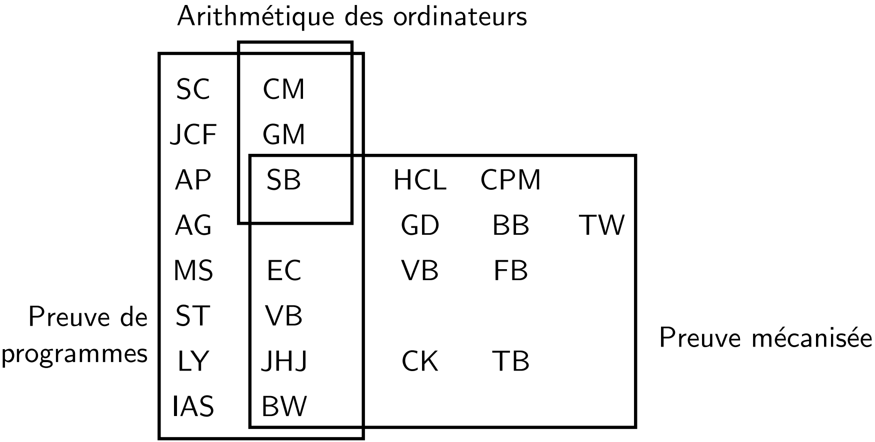
CM
GM
SB

# structure

Arithmétique des ordinateurs

| | | |
|---|---|---|
| | SC | CM |
| | JCF | GM |
| | AP | SB |
| | AG | |
| | MS | EC |
| Preuve de programmes | ST | VB |
| | LY | JHJ |
| | IAS | BW |

# structure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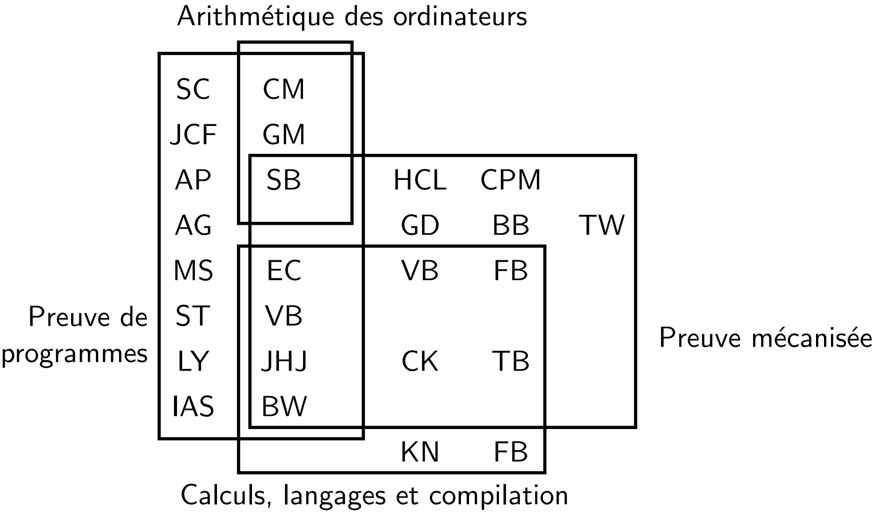
Preuve de programmes
Preuve mécanisée

# structure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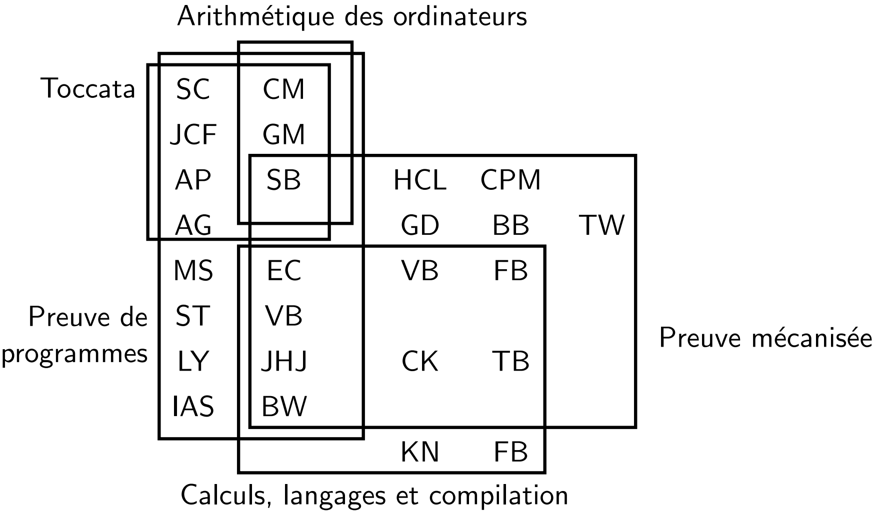
Calculs, langages et compilation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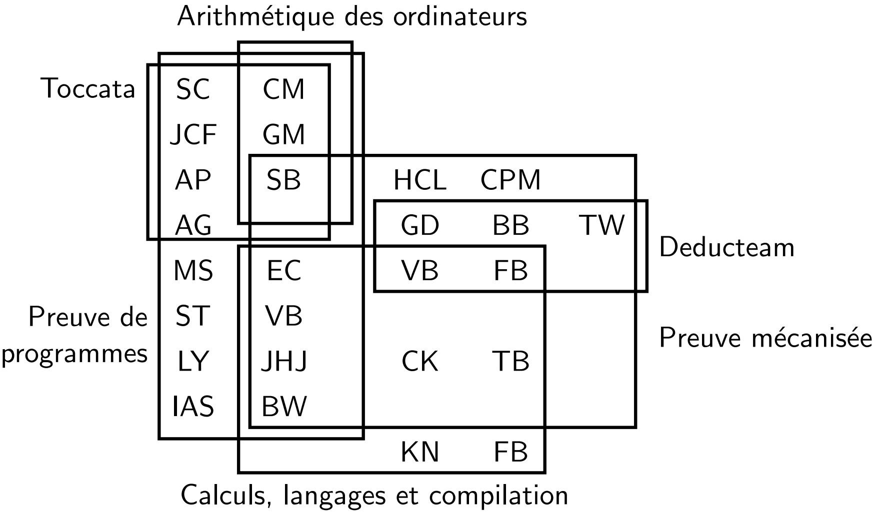
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BW
KN
FB
Calculs, langages et compilation

Responsable : Guillaume Melquiond

- IEEE 754 floating-point numbers
- GMP-like big integers
- interval arithmetic

✍ Sylvie Boldo, Guillaume Melquiond.
*Some Formal Tools for Computer Arithmetic : Flocq and Gappa.*
ARITH 2021

Sylvie Boldo co-leader GDR IM's GT ARITH

Responsable : Kim Nguyễn

- formal semantics
- study and design of languages
- programming and domain-specific languages

✍ Valentin Blot.
*A direct computational interpretation of second-order arithmetic via update recursion*. LICS 2022.

Responsable : Andrei Paskevich

- deductive program verification
- reasoning on mutable memory
- verification of properties of systems

✍ Cláudio Belo Lourenço Denis Cousineau, Florian Faissole, Claude Marché, David Mentré, Hiroaki Inoue.
*Automated Verification of Temporal Properties of Ladder Programs*. Formal Methods for Industrial Critical Systems 2021.

Responsable : Chantal Keller

- proof systems
- interaction between proof systems
- formalized libraries

✍ Frédéric Blanqui, Gilles Dowek, Émilie Grienenberger, Gabriel Hondet, François Thiré.
*Some Axioms for Mathematics*. FSCD 2021

# Ongoing grants

## International

- EMC2 [ERC Synergy →2025]
- EuroProofNet [COST →2025]
- FRESCO [ERC Consolidator →2026]

## National

- France–Japan [Sakura →2023]
- Gospel [ANR →2026]
- ICSPA [ANR →2024]
- NARCO [ANR →2026]
- NuSCAP [ANR →2025]
- SecurEval
  [PEPR Cybersécurité →2027]

## Industrial

- AdaCore [bilatéral →2023]
- Langages dynamiques pour les données
  [Oracle Labs →2022]
- Mitsubishi Electric R&D
  [bilatéral →2023]
- TrustInSoft [bilatéral →2023]
- thèses CIFRE
  [OCamlPro, Tarides →2023]

Mainly at LMF :

- Alt-Ergo
- CoqInterval
- Coq-num-analysis
- Coquelicot
- CDuce
- Creusot
- Cubicle
- Datacert
- Datalogcert
- Dedukti
- Ekstrakto
- Flocq
- Gappa
- Lambdapi
- OntoEventB
- SMTCoq
- Sniper
- Why3
- WhyMP

LMF also contributes to :

- Coq
- HOL-TestGen
- Iris
- Isabelle
- OCaml

# PhD students

In preparation :

- Léo Andrès [+OCamlPro]
- Luc Chabassier
- Xavier Denis
- Louise Dubois De Prisque
- Thiago Felicissimo
- Valentin Fouillard [+LISN]
- Paul Geneau de Lamarlière [+MERCE]
- Yoan Géran
- Emilie Grienenberger
- Baptiste Gueuziec [+CEA]
- Alexandrina Korneva
- Antoine Lanco
- Mickael Laurent [+U Paris Cité]
- Amélie Ledein
- Nicolas Meric
- Glen Mevel [+Inria]
- Josué Moreau
- Houda Mouhcine [+LIPN +Inria Paris]
- Clément Pascutto [+Tarides]
- Quentin Petitjean [+LIG]
- Andrew Samokhish [+K. Inside]
- Julien Simonnet [+CEA]
- Rébecca Zucchini

Defended :

- Yacine El Haddad [09/21]
- Gaspard Ferey [06/21]
- Diane Gallois-Wong [+LIP6] [03/21]
- Quentin Garchery [01/22]
- Guillaume Girol [+CEA] [10/22]
- Gabriel Hondet [09/22]
- Yaëlle Vinçont [12/21, +CEA]

# Proof of programs with WHY3 and CREUSOT

Andrei Paskevich    Jacques-Henri Jourdan

LMF — October 19, 2022

# Deductive program verification

```
(*  Given an array a of signed integers, compute the maximum sum s *)
(*  of its contiguous subarray. The sum of an empty subarray is 0. *)


val maximum_subarray (a: array int): (s: int)
```

```
(*  Given an array a of signed integers, compute the maximum sum s *)
(*  of its contiguous subarray. The sum of an empty subarray is 0. *)



let maximum_subarray (a: array int): (s: int)
=
  let ref max = 0 in
  let ref cur = 0 in
  for i = 0 to length a - 1 do
    cur += a[i];
    if cur < 0 then cur <- 0;
    if cur > max then max <- cur;
  done;
  return max
```

```
(*  Given an array a of signed integers, compute the maximum sum s *)
(*  of its contiguous subarray. The sum of an empty subarray is 0. *)

let maximum_subarray (a: array int): (s: int)
  ensures { forall l h: int. 0 <= l <= h <= length a -> sum a l h <= s }
  ensures { exists l h: int. 0 <= l <= h <= length a /\ sum a l h  = s }
=
  let ref max = 0 in
  let ref cur = 0 in
  for i = 0 to length a - 1 do
    cur += a[i];
    if cur < 0 then cur <- 0;
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  for i = 0 to length a - 1 do






    cur += a[i];
    if cur < 0 then ( cur <- 0 );
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  ensures { exists l h: int. 0 <= l <= h <= length a /\ sum a l h  = s }
=
  let ref max = 0 in
  let ref cur = 0 in
  let ghost ref cl = 0 in
  let ghost ref lo = 0 in
  let ghost ref hi = 0 in
  for i = 0 to length a - 1 do
    invariant { forall l: int. 0 <= l <= i -> sum a l i <= cur }
    invariant { 0 <= cl <= i /\ sum a cl i = cur }
    invariant { forall l h: int. 0 <= l <= h <= i -> sum a l h <= max }
    invariant { 0 <= lo <= hi <= i /\ sum a lo hi = max }
    cur += a[i];
    if cur < 0 then ( cur <- 0; cl <- i+1 );
    if cur > max then ( max <- cur; lo <- cl; hi <- i+1 )
  done;
  return max
```
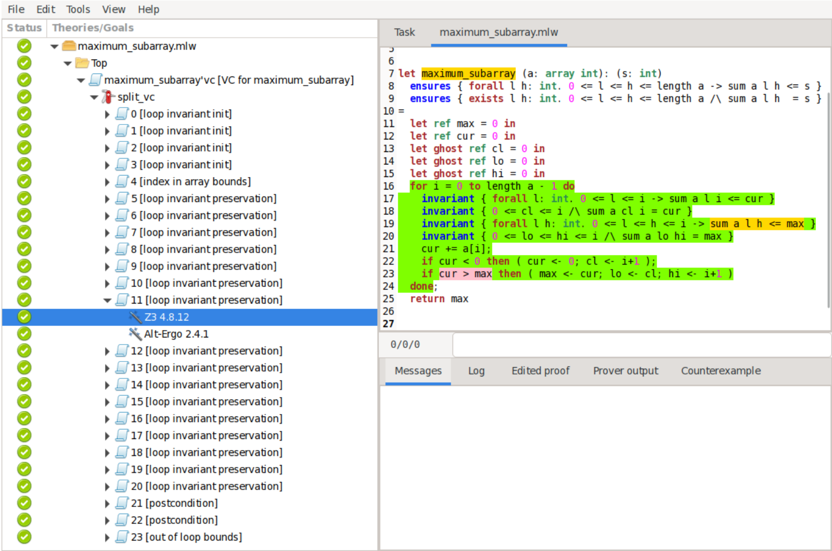

# WHY3 proof session



# WHY3 in a nutshell

WHYML, a programming language

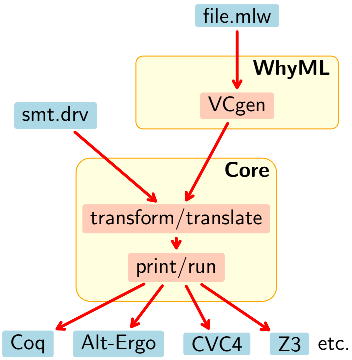
- type polymorphism • variants
- limited support for higher order
- pattern matching • exceptions
- integer & FP computer arithmetic
- mutable data with controlled aliasing
- ghost code and ghost data

WHY3, a program verification tool

- VC generation using WP or SP
- 70+ VC *transformations* ($\approx$ tactics)
- support for 25+ ATP and ITP systems
- produces OCaml and C code

WHYML, a specification language

- polymorphic & algebraic types
- limited support for higher order
- inductive predicates



Three different ways of using WHY3

- as a logical language
  - a convenient front-end to many theorem provers
  - probabilistic programs: EASYCRYPT (Barthe et al.)
- as a programming language for correct-by-construction software
  - see examples in our gallery
    `https://toccata.gitlabpages.inria.fr/toccata/gallery/why3.en.html`
  - Multiple Precistion arithmetic library: WHYMP (Melquiond Rieu-Helft)
- as an intermediate verification language
  - Ada programs: SPARK (AdaCore)
  - C programs: FRAMA-C (Marché Moy)
  - Java programs: KRAKATOA (Marché Paulin Urbain)
  - Rust programs: CREUSOT (Denis Jourdan Marché)

# WhyMP: arbitrary-precision integer arithmetic

The GNU Multiple Precision arithmetic library (GMP)

- free software, widely used
- state-of-the-art algorithms, unmatched performances

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- highly intricate algorithms written in low-level C and ASM
- ill-suited for random testing

  GMP 5.0.4: *"Two bugs in multiplication [. . . ] with extremely low probability [. . . ]. Two bugs in the gcd code [. . . ] For uniformly distributed random operands, the likelihood is infinitesimally small."*

# WhyMP: arbitrary-precision integer arithmetic

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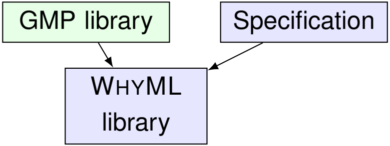
## Objectives

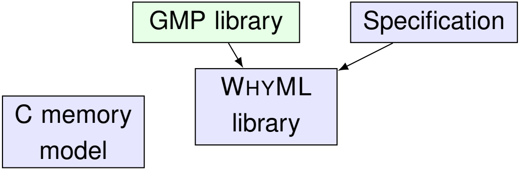
- produce a verified library compatible with GMP
- attain performances comparable to a no-assembly GMP

GMP library

# The WHY3 workflow



# The WHY3 workflow



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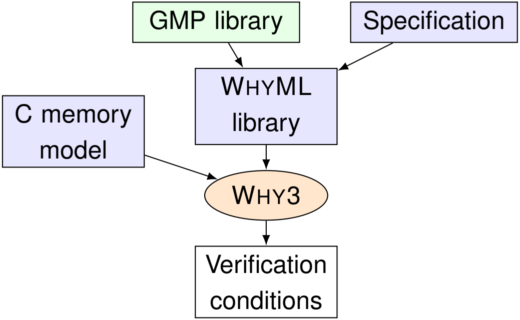

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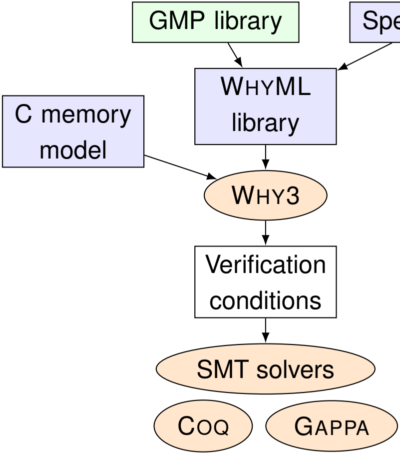
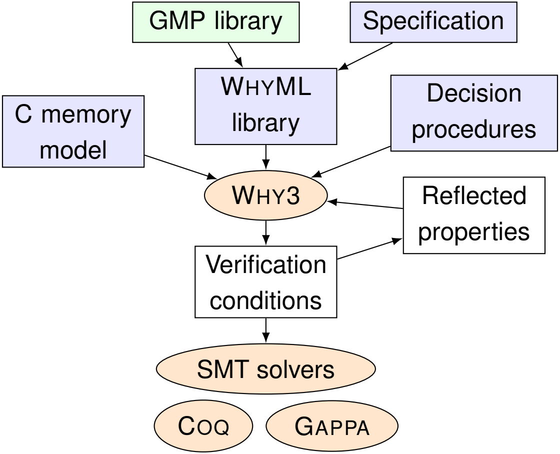


# The WHY3 workflow



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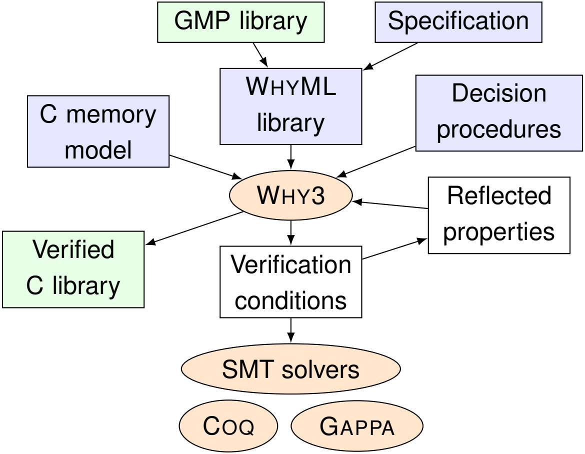

# Example: subtracting 1 from a long integer

Original macro (simplified from 18-line mpn_decr_u)

```
#define mpn_decr_1(p)          \
  mp_ptr __p = (p);            \
  while ((*(__p++))-- == 0) ;
```

# Example: subtracting 1 from a long integer

## Original macro (simplified from 18-line mpn_decr_u)

```
#define mpn_decr_1(p)          \
  mp_ptr __p = (p);            \
  while ((*(__p++))-- == 0) ;
```

## Conversion to WhyML

```
let wmpn_decr_1 (p: ptr uint64) : unit
=
  let ref lp = 0 in
  while lp = 0 do
    lp <- get p;
    set p (sub_mod lp 1);
    p <- p ++ 1;
  done
```

# Example: subtracting 1 from a long integer

## Original macro (simplified from 18-line `mpn_decr_u`)

```
#define mpn_decr_1(p)          \
  mp_ptr __p = (p);            \
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```

## Conversion to WhyML

```
let wmpn_decr_1 (p: ptr uint64) (ghost sz: int32) : unit
  requires { valid p sz }
  requires { 1 <= value p sz } (* <- real bug in GMP *)
  ensures  { value p sz = value (old p) sz - 1 }
=
  let ref lp = 0 in
  while lp = 0 do
    lp <- get p;
    set p (sub_mod lp 1);
    p <- p ++ 1;
  done
```

# Example: subtracting 1 from a long integer

## Original macro (simplified from 18-line mpn_decr_u)

```
#define mpn_decr_1(p)          \
  mp_ptr __p = (p);            \
  while ((*(__p++))-- == 0) ;
```

## Extraction to C

```
void wmpn_decr_1(uint64_t * p) {
  uint64_t lp = 0;
  while (lp == 0) {
    lp = *p;
    *p = lp - 1;
    p = p + 1;
  }
}
```

# A Verified Library

22 000 lines of WhyML code — 5 000 lines of extracted C code

Performance: comparable to GMP without assembly code

Supported operations:

- addition, subtraction, comparison
- multiplication: quadratic, Toom-Cook 2, Toom-Cook 2.5
- division: “schoolbook”
- square root: divide-and-conquer
- modular exponentiation with Montgomery reduction
- base conversion and input-output

# A new systems programming language: Rust

High performance, high level of control with safe abstraction mechanisms and concurrency features.

Tends to replace C/C++ for many sensitive applications:

- Web browsers (Firefox),
- Cloud infrastructures (Amazon),
- Operating systems kernels (Linux),
- Embedded systems

How to guarantee correctness of Rust code?

# Our claim

Rust is well-suited for verification.

It has a rich type system.

- Safety guarantees.
  - No need to prove safety.
- Non-aliasing guarantees.
  - Helps reasoning with pointers.
- Type traits (i.e., type classes): abstraction and genericity mechanism.
  - Easy to write reusable specifications.

Creusot, a deductive verification tool for Rust.

- The user annotates Rust code with specifications.
- Creusot translates Rust into WhyML (Why3 = Creusot's back-end).
- Exploits type system (non-aliasing, genericity with traits, safety).

One crucial aspect of Creusot: lightweight handling of pointers.

Our big secret:

**Rust is a purely functional* programming language**

... and this greatly helps verification.

*some squinting required.

```rust
fn incr(mut x: u64, y: u64) -> (u64, u64) {
  x += y;
  do_stuff();
  (x, y)
}
```

```ocaml
let incr x y =
  let x = x + y in
  do_stuff ();
  (x, y)
```

As long as there is no pointer, mutating variables = shadowing.

We give a name to the value of each variable at each program point.
Variables can only be mutated directly, we can track their modifications.

`Box<T>` is Rust's type for owned pointers.

- I.e., pointers that are allocated on the heap ($\simeq$ `malloc`).
- Type system guarantee: no alias.

```
fn incr(x: Box<u64>, y: Box<u64>)
      -> (Box<u64>, Box<u64>) {
  *x += *y;
  do_stuff();
  (x, y)
}
```

```
let incr x y =
  let x = x + y in
  do_stuff();
  (x, y)
```

Because there is no alias, we know nobody else can mutate `x` and `y`.

Owned pointers functionally behave like their content.

Rust has another notion of pointer: borrows.

- Temporary pointer to a memory location.
- They are crucial to effective use of Rust.

Their handling in Creusot requires a new mechanism: prophecies.

- Prophecies allow concise specifications.
- But their soundness cause subtle causality issues.
  - We have a strong meta-theoretical formalization.

# Occurence Typing and Set-theoretic types for dynamic languages

Kim NGUYỄN

19 octobre 2022

# Motivations

## A function in JavaScript

```
function foo(x) {
  return (typeof(x) === "number")? x+1 : x.trim();
  }
```

Idiomatic in dynamic languages:

- JavaScript: optional arguments (`typeof(y) === "undefined"`)
- JavaScript: poor person's overloading
- Python: 5400 dynamic type tests in the standard library
- Python: `isinstance(x,(int, float, str))`

# Motivations

## A function in TypeScript (Microsoft)/Flow (Facebook)

```
function foo(x : number | string) : number | string {
  return (typeof(x) === "number")? x+1 : x.trim();
  }
```

# Motivations

## A function in TypeScript (Microsoft)/Flow (Facebook)

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function foo(x : number | string) : number | string {
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```

$(\texttt{number|string}) \rightarrow (\texttt{number|string})$

# Motivations

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function foo(x : number | string) : number | string {
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```

$$(\texttt{number|string}) \to (\texttt{number|string})$$

We present a type system with occurrence typing and:

# Motivations

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```

(number|string) → (number|string)

We present a type system with occurrence typing and:

- Expressive types: (number → number) ∧ (string → string) (e.g., `foo(42)+1`)

# Motivations

## A function in TypeScript (Microsoft)/Flow (Facebook)

```
function foo(x : number | string) : number | string {
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$(\texttt{number|string}) \to (\texttt{number|string})$

We present a type system with occurrence typing and:

- Expressive types: $(\text{number} \to \text{number}) \wedge (\text{string} \to \text{string})$ (e.g., `foo(42)+1`)
- Inference (no need for user-defined type annotations)

# Motivations

## A function in JavaScript

```
function foo(x) {
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- Inference (no need for user-defined type annotations)

# Motivations

## A function in JavaScript

```
function foo(x) {
  return (is_number_or_function(x))? x+1 : x.trim();
  }
```

$(\texttt{number|string}) \to (\texttt{number|string})$

We present a type system with occurrence typing and:

- Expressive types: $(\text{number} \to \text{number}) \wedge (\text{string} \to \text{string})$ (e.g., `foo(42)+1`)
- Inference (no need for user-defined type annotations)
- Works for checks with arbitrary expressions (*eg*, applications of user-defined functions)

# Language

## Functional language

$$
\begin{array}{rrcl}
\textbf{Types} & t & ::= & \mathsf{Int} \mid \mathsf{Bool} \mid t \to t
\end{array}
$$

$$
\begin{array}{rrcl}
\textbf{Expressions} & e & ::= & c \mid x \mid \lambda x.e \mid ee \\
\textbf{Values} & v & ::= & c \mid \lambda x.e
\end{array}
$$

Call-by-value semantics:

$$(\lambda x.e)v \quad \rightsquigarrow \quad e\{v/x\}$$

# Language

## Functional language with set-theoretic types

$$\textbf{Types} \quad t \quad ::= \quad \mathrm{Int} \mid \mathrm{Bool} \mid t \to t \mid t \vee t \mid t \wedge t \mid \neg t \mid \mathtt{Any}$$

$$\begin{array}{rccl} \textbf{Expressions} & e & ::= & c \mid x \mid \lambda x.e \mid ee \\ \textbf{Values} & v & ::= & c \mid \lambda x.e \end{array}$$

Call-by-value semantics:

$$(\lambda x.e)v \quad \rightsquigarrow \quad e\{v/x\}$$

# Language

Functional language with set-theoretic types and typecases

$$\textbf{Types} \quad t \quad ::= \quad \mathsf{Int} \mid \mathsf{Bool} \mid t \to t \mid t \vee t \mid t \wedge t \mid \neg t \mid \mathtt{Any}$$

$$\begin{array}{rcll} \textbf{Expressions} & e & ::= & c \mid x \mid \lambda x.e \mid ee \mid (e{\in}t)\,?\,e : e \\ \textbf{Values} & v & ::= & c \mid \lambda x.e \end{array}$$

Call-by-value semantics:

$$(\lambda x.e)v \quad \rightsquigarrow \quad e\{v/x\}$$

# Language

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Call-by-value semantics:

$$\begin{array}{rcll} (\lambda x.e)v & \leadsto & e\{v/x\} & \\ (v{\in}t)\,?\,e_1 : e_2 & \leadsto & e_1 & \text{if } v \in t \\ (v{\in}t)\,?\,e_1 : e_2 & \leadsto & e_2 & \text{if } v \notin t \end{array}$$

# Type System

## Occurrence Typing and Typecases Part

$$[\in_1]\ \frac{\Gamma \vdash e : t \qquad \Gamma \vdash e_1 : t_1}{\Gamma \vdash (e{\in}t)\ ?\ e_1 : e_2 : t_1} \qquad [\in_2]\ \frac{\Gamma \vdash e : \neg t \qquad \Gamma \vdash e_2 : t_2}{\Gamma \vdash (e{\in}t)\ ?\ e_1 : e_2 : t_2}$$

$$[\vee]\ \frac{\Gamma \vdash e' : t_1 \vee t_2 \qquad \Gamma, x : t_1 \vdash e : t \qquad \Gamma, x : t_2 \vdash e : t}{\Gamma \vdash e\{e'/x\} : t}$$

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### Rationale

- $[\vee]$ splits the type of the checked expression as necessary
- $[\in_i]$ distribute the appropriate split to each branch

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### Rationale (i.e., the essence of occurrence typing)

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**Rationale (i.e., the essence of occurrence typing)**

- $[\vee]$ splits the type of the checked expression as necessary
- $[\in_i]$ distribute the appropriate split to each branch

Example: For $f : (\mathrm{Int} \to (\mathrm{Int} \vee \mathrm{Bool})) \wedge (\neg\mathrm{Int} \to \mathrm{Bool})$

$\lambda x.(f\ x{\in}\mathrm{Int})\ ?\ (f\ x) + x + 1 : 42$ has type $(\mathrm{Int}{\to}\mathrm{Int}) \wedge (\neg\mathrm{Int}{\to}42)$

# Towards an algorithmic system

## What is not algorithmic in $[\vee]$?

$$[\vee]\ \frac{\Gamma \vdash e' : t_1 \vee t_2 \qquad \Gamma, x : t_1 \vdash e : t \qquad \Gamma, x : t_2 \vdash e : t}{\Gamma \vdash e\{e'/x\} : t}$$

- It can be applied anywhere (on any expression)
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Not syntax directed
$\Rightarrow$ *Normal forms*

- It can use any decomposition of the type of $e'$

Not analytic
$\Rightarrow$ Annotations

# Maximal-Sharing Canonical Form

[$\simeq$ A-normal form on steroids, used for typing only]

- Apply [$\vee$] on **every** subexpression
  $\Rightarrow$ Define each subexpression in a separate definition
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  $\Rightarrow$ Two occurrences of the same expression share the **same definition**

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$$
\begin{array}{l}
MSCF(\lambda x.(f\ x{\in}\mathsf{Int})\,?\,((f\ x)+x+1):42) = \\
\qquad \lambda x.\begin{array}[t]{l}
\texttt{bind}\ u = x+1\ \texttt{in} \\
\texttt{bind}\ v = f\,x\ \texttt{in} \\
\texttt{bind}\ w = v+u\ \texttt{in} \\
\texttt{bind}\ y = (v{\in}\mathsf{Int})\,?\,w:42\ \texttt{in} \\
y
\end{array}
\end{array}
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| | | | |
|---|---|---|---|
| **Atomic expressions** | $\boldsymbol{a}$ | $::=$ | $c \mid \lambda x.\boldsymbol{\kappa} \mid (x, x) \mid xx \mid (x{\in}\tau)\,?\,x : x \mid$ |
| **Canonical Forms** | $\boldsymbol{\kappa}$ | $::=$ | $x \mid \texttt{bind}\, x = \boldsymbol{a}\ \texttt{in}\ \boldsymbol{\kappa}$ |

[$\vee$] now becomes:

$$[\vee]\ \frac{\Gamma \vdash e' : t_1 \vee t_2 \qquad \Gamma, x : t_1 \vdash e : t \qquad \Gamma, x : t_2 \vdash e : t}{\Gamma \vdash \texttt{bind}\, x = e'\ \texttt{in}\ e\ : t}$$

# Annotations

## Example

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\begin{array}{l}
MSCF(\lambda x.(f\ x{\in}\mathsf{Int})\ ?\ ((f\ x) + x + 1) : 42) = \\
\qquad \lambda x : \{\mathsf{Int}, \neg\mathsf{Int}\}\ . \\
\qquad\quad \mathtt{bind}\ u : \{(x{:}\mathsf{Int}){\triangleright}\mathsf{Int}\} = x + 1\ \mathtt{in} \\
\qquad\quad \mathtt{bind}\ v : \{\mathsf{Int}, \mathsf{Bool}\} = f\ x\ \mathtt{in} \\
\qquad\quad \mathtt{bind}\ w : \{(u{:}\mathsf{Int}, v{:}\mathsf{Int}){\triangleright}\mathsf{Int}\} = v + u\ \mathtt{in} \\
\qquad\quad \mathtt{bind}\ y : \{\mathsf{Int}, 42\} = (v{\in}\mathsf{Int})\ ?\ w : 42\ \mathtt{in} \\
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$$\begin{array}{l} MSCF(\lambda x.(f\ x{\in}\mathsf{Int})\ ?\ ((f\ x)+x+1):42) = \\ \quad \lambda x : \{\mathsf{Int}, \neg\mathsf{Int}\}\,. \\ \qquad \mathtt{bind}\ u : \{(x{:}\mathsf{Int}){\triangleright}\mathsf{Int}\} = x+1\ \mathtt{in} \\ \qquad \mathtt{bind}\ v : \{\mathsf{Int}, \mathsf{Bool}\} = f\ x\ \mathtt{in} \\ \qquad \mathtt{bind}\ w : \{(u{:}\mathsf{Int}, v{:}\mathsf{Int}){\triangleright}\mathsf{Int}\} = v+u\ \mathtt{in} \\ \qquad \mathtt{bind}\ y : \{\mathsf{Int}, 42\} = (v{\in}\mathsf{Int})\ ?\ w:42\ \mathtt{in} \\ \qquad y \end{array}$$

*Results:*

- $e$ typable $\iff$ $MSCF(e)$ typable $\iff$ annotated-$MSCF(e)$ typable
- Transforming into MSCF is effective
- Type-checking for annotated MSCFs is decidable

# Annotations

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**Is $e$ typable?**

If possible to annotate $MSCF(e)$ to give it some type $t$, then $\vdash e : t$.
Otherwise, $e$ is not typable.

# Annotations

## Example

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*Results:*

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### Is $e$ typable?

**We defined and implemented a sound algorithm that determines annotations using an analysis of the applications and typecases**

# Examples from our prototype

```
type Falsy = False | "" | 0
type Truthy = ~Falsy
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                 else true
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let to_Bool x = not_ (not_ x)
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$$\left.\right\}\begin{array}{l}(\mathsf{Falsy}\to\mathsf{Any}\to\mathsf{False})\wedge \\ (\mathsf{Truthy}\to\mathsf{Truthy}\to\mathsf{True})\wedge \\ (\mathsf{Truthy}\to\mathsf{Falsy}\to\mathsf{False})\end{array}$$

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# Current and future work

This work (collaboration with Giuseppe Castagna (IRIF) and Mickaël Laurent (IRIF & LMF)) has been presented at POPL22. Since then

- Polymorphism:

```
concat : [ 'a * ] -> [ 'b* ] -> [ 'a* 'b* ]
filter : (('a & 'b) -> True & ('a \ 'b) -> False) -> ['a*] ->
     [('a&'b)*]
```

- Started work on integrating side effects (with Thibaut Balabonski and Philippe Volte–Viera, PEPR Secureval).