Mechanized verification of type systems using Iris

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The old problem of proving “type safety”:

“Well-typed programs cannot go wrong”

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Goal of this talk:

- Introduce the "logical approach" in separation logic as an alternative to the standard progress/preservation approach to type safety
- Show that this approach is well-suited for mechanization of challenging substructural type systems (*e.g.*, session types and Rust) in Rocq
- Show that this approach makes it possible to type "unsafe" code

Recap: Progress and preservation [Wright and Felleisen, simplified by Harper]

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2. **Preservation:** If $\vdash e : A$ and $e \to e'$ then $\vdash e' : A$

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Proof of type safety: If $\vdash e : A$ then $\mathsf{safe}(e)$
Obtain $\vdash e' : A$ by induction on length of $e \to^* e'$ and preservation,
conclude by progress

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  Disjointness conditions show up everywhere
  (And Rocq does not accept "left as an exercise for the reader")

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- Even more tricky once you consider a substructural type system
  Disjointness conditions show up everywhere
  (And Rocq does not accept "left as an exercise for the reader")
- Unsuitable to reason about "unsafe" code
  `unsafe` in Rust, `Obj.magic` in OCaml, `unsafePerformIO` in Haskell

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Key challenge: Define $\vDash e : A$ so that:

- It is rich enough to support challenging PL features
- It allows for a concise proof of the fundamental theorem

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- More abstract versions developed by Appel *et al.* (2007) and Dreyer *et al.* (2011)
- Iris provides a modern **logical approach** in which concurrent separation logic hides reasoning about state *and* which is well-suited for mechanized proofs in Rocq

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And then change some conjunctions into separation conjunctions to scale to a substructural type system

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application is not a value, we need to talk about its result

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Weakest precondition:

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closing substitution, I will ignore those most of the time

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Proofs of key properties

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The proof is streamlined through abstract reasoning rules about WP, thereby avoiding explicit reasoning abut the operational semantics

An "unsafe" fixpoint combinator

Consider a strict version of Curry's fixpoint operator:

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✓ Yes. We can prove that **fix** is semantically safe

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Important observations:

- Each type is given a semantic interpretation via the Curry-Howard correspondence
- Instead of Rocq's Prop we can use **a logic with more fancy connectives** to interpret challenging types (*e.g.*, substructural types)

higher-order concurrent separation logic

A more feature-rich logical relation

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Substructural types

Intuition and simple typing rules

Variables can be used **exactly (linear)** or **at-most (affine)** once

For example, $\lambda f.\, \lambda x.\, f\ x\ x$ is **not typeable**

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Useful when types denote ownership of resources

- Session types: Channels – Ensure protocol compliance
- Rust: Memory locations – Avoid use after free and data races

Semantic typing for a substructural type system

Semantic interpretation of types:

$$[\![_]\!] : \mathsf{Type} \to \mathsf{SemType} \quad \text{where} \quad \mathsf{SemType} \triangleq \mathsf{Val} \to \mathsf{sepProp}$$

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Iris **invariant** $\boxed{P}$ $\approx$ knowledge that P holds at all times (invariantly)

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This scales—pick the right Iris features to interpret your favorite types

Summary: Recipe for verifying a type system in Iris

1. Define the syntax and operational semantics for your language
2. Build a program logic using Iris, *i.e.*, define WP, $\mapsto$, *etc.*
3. Verify separation logic APIs for your "unsafe" libraries
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5. Prove semantic typing rules/fundamental theorem
6. Profit

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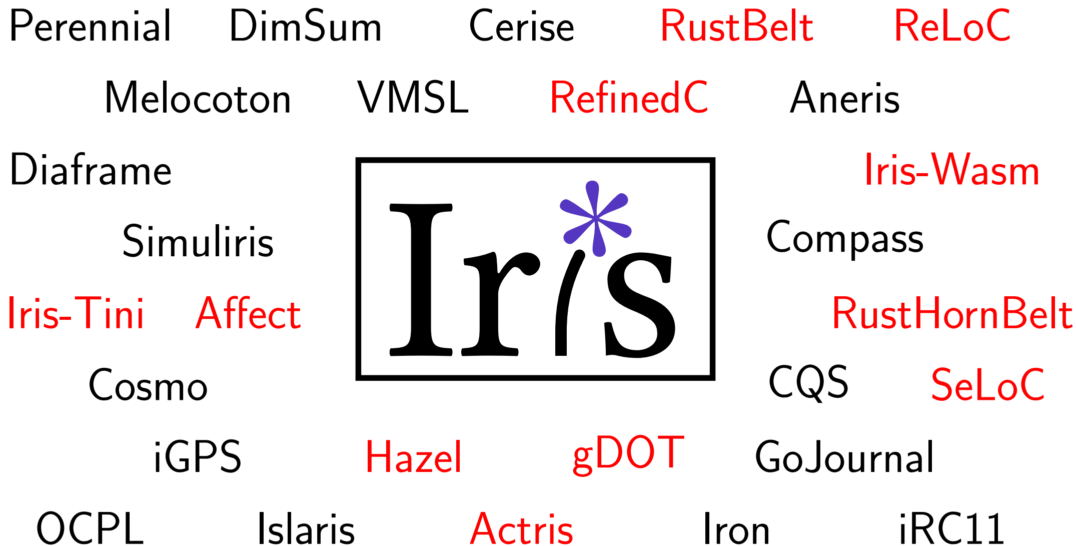
   Interpret type formers using suitable logical connectives through Curry-Howard
5. Prove semantic typing rules/fundamental theorem
   Most of the heavy lifting is done by the Hoare/WP rules in Iris
6. Profit

The logical approach in Iris scales



Future work: Going beyond safety

- Applying the logical approach to deadlock freedom, resource leak freedom, liveness, non-interference remains challenging
- Different models of concurrent separation logic/Iris need to be explored: linear (instead of affine), transfinite, *etc.*
- We have initial versions for specific languages
- But we do not have the right Iris-style abstractions to build these logics modularly
- Nor to easily combine different PL features in one type safety proof