Weighted NetKAT

A Programming Language for Quantitative Network Verification

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Collaborating with Galois and STR

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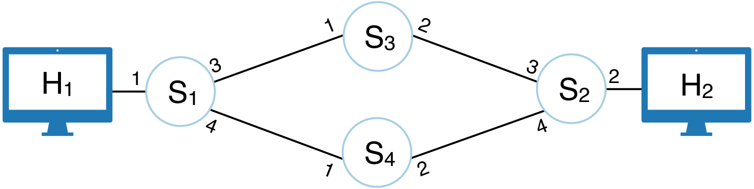
April, 2026

Classic NetKAT [Foster et al. '14]

packet

dst	sw	pt	$\cdots$
H_2	S_1	3	$\cdots$



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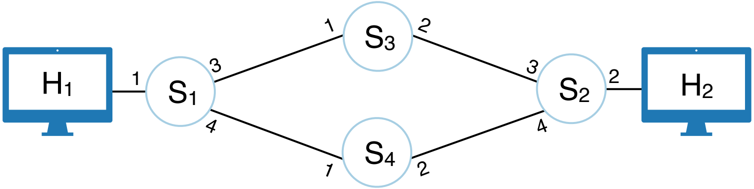

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

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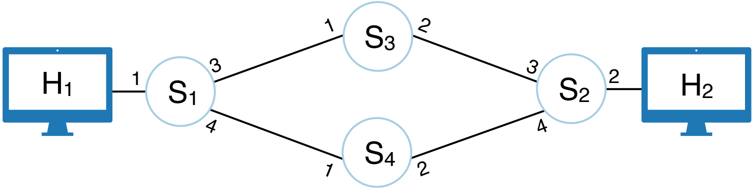

$$in\,; (policy\,; link\,; \mathsf{dup})^*\,; out$$

$$in, out \quad \triangleq \quad (\mathsf{sw} = S_1 \wedge \mathsf{pt} = 1) \vee (\mathsf{sw} = S_2 \wedge \mathsf{pt} = 2)$$

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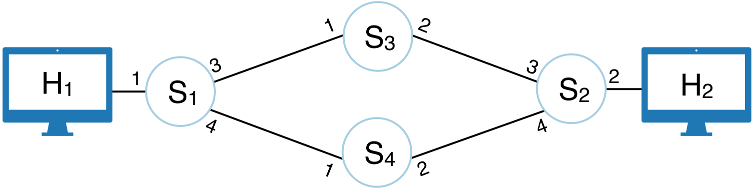

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

$$policy \triangleq \text{if}\,\mathsf{dst} = H_2 \wedge \mathsf{sw} = 1\,\text{then}\,(\mathsf{pt} \leftarrow 3 \oplus \mathsf{pt} \leftarrow 4)\,\text{else}\ldots$$

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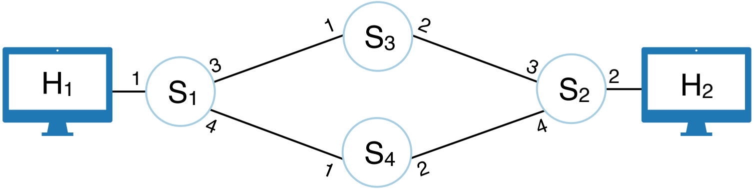

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

$$link \quad \triangleq \quad \mathsf{if\,sw} = 1 \wedge \mathsf{pt} = 3\,\mathsf{then\,sw} \leftarrow S_3\,;\mathsf{pt} \leftarrow 1\,\mathsf{else}\ \ldots$$

Classic NetKAT [Foster et al. '14]

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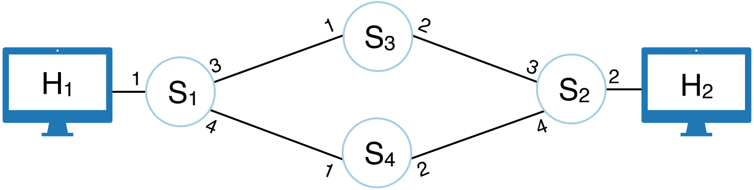

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

"Can H_1 reach H_2?"

Weighted NetKAT

packet

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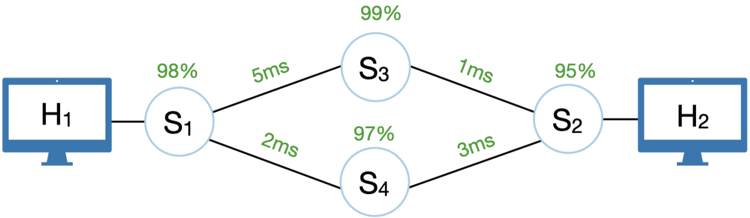

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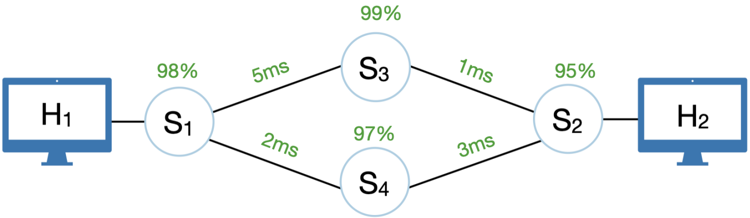

"Do all paths from H_1 to H_2
take $\leq$ 5ms and succeed with probability $\geq$ 90%?"

packet

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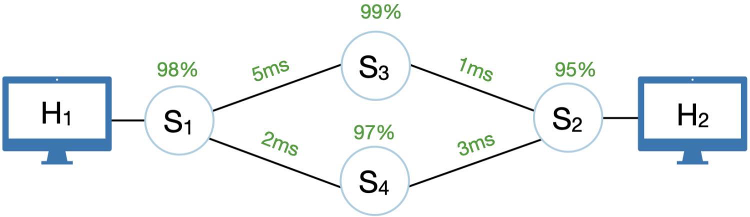
H_2	S_1	3	...



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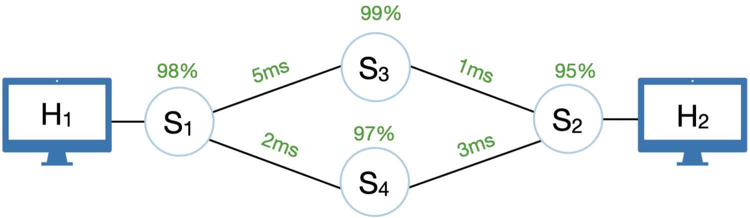

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

$$policy \triangleq \mathsf{if}\,\mathsf{dst} = H_2 \wedge \mathsf{sw} = 1\,\mathsf{then}\,98\% \odot (\mathsf{pt} \leftarrow 3 \oplus \mathsf{pt} \leftarrow 4)\,\mathsf{else}\ldots$$

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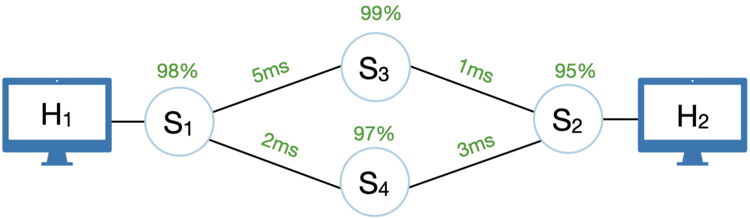

$$in\,;(policy\,;link\,;\mathsf{dup})^*\,;out$$

$$link \triangleq \text{if}\,\mathsf{sw} = 1 \wedge \mathsf{pt} = 3\,\text{then}\,\mathbf{5ms} \odot (\mathsf{sw} \leftarrow S_3\,;\mathsf{pt} \leftarrow 1)\,\text{else}\ldots$$

Tropical semiring to model **Confidentiality** or **Cost** $(\mathbb{N}^\infty, \min, +, \infty, 0)$

Weighting $r \odot p$:
Policy p reveals r bits of information (or has cost r)

Choice $p \oplus q$:
Choose whichever policy reveals less bits of information (or has cheaper cost)

Interpretation of $[\![p]\!](h)$:
Best-case confidentiality (or cost) between two hosts

Bottleneck semiring to model **Network Throughput** $(\mathbb{N}^\infty_{-\infty}, \max, \min, -\infty, \infty)$

Weighting $r \odot p$:
Restrict throughput of policy p to r mbps

Choice $p \oplus q$:
Choose policy with higher throughput

Interpretation of $[\![p]\!](h)$:
Best-case network throughput between two hosts

Security semiring to model **Security Levels** $(0 < \mathrm{L} < \mathrm{M} < \mathrm{H}, \max, \min, 0, \mathrm{H})$

Weighting $r \odot p$:
Policy p has security level r

Choice $p \oplus q$:
Choose policy with higher security level

Interpretation of $[\![p]\!](h)$:
Best-case security level between two hosts

Viterbi semiring to model **Reliability** $([0, 1], \max, \cdot, 0, 1)$

Weighting $r \odot p$:
Policy p has a success rate of r

Choice $p \oplus q$:
Choose policy with higher success rate

Interpretation of $[\![p]\!](h)$:
Best-case reliability between two hosts

and more ...

Classic NetKAT: Syntax and Semantics

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Packets $\pi\colon \{f_1, \ldots, f_m\} \to \mathbb{N}$

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$$\begin{aligned} & [\![f \leftarrow 1 \oplus f \leftarrow 2]\!](\pi) \\ = \ & \{\pi[f \mapsto 1], \\ & \pi[f \mapsto 2]\} \end{aligned}$$

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ω-continuous semiring $\mathcal{S} = (S, \oplus, \odot, \mathbf{0}, \mathbf{1})$

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$$[\![p]\!] : \mathsf{Pkts} \to (\mathsf{Hists} \to S)$$

$$[\![(\mathbf{3} \odot f \leftarrow 1) \oplus (\mathbf{5} \odot f \leftarrow 2)]\!](\pi)$$

$$= \lambda h. \begin{cases} \mathbf{3} & \text{if } h = \pi[f \mapsto 1] \\ \mathbf{5} & \text{if } h = \pi[f \mapsto 2] \\ \mathbf{0} & \text{o.w.} \end{cases}$$

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$$[\![(\mathbf{3} \odot f \leftarrow 1) \oplus (\mathbf{5} \odot f \leftarrow 1)]\!](\pi)$$

$$= \lambda h. \begin{cases} \mathbf{5} & \text{if } h = \pi[f \mapsto 1] \\ \mathbf{0} & \text{o.w.} \end{cases}$$

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Decidability & Computability Results

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Let $\mathcal{S} = (S, \oplus, \odot, \mathbf{0}, \mathbf{1})$ be a computable semiring.

Theorem

$[\![p]\!]$ is a computable function.

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Idea: Construct **Weighted NetKAT Automaton** $\mathcal{A}_p$ with semantics (simplified)

$$\underbrace{[\![\mathcal{A}_p]\!] \colon \mathsf{Hists} \to S}_{\text{regular (rational) weighted language of histories}}$$

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$$\underbrace{[\![\mathcal{A}_p]\!] \colon \mathsf{Hists} \to S}_{\text{regular (rational) weighted language of histories}}$$

such that

$$\forall \pi \colon \forall h \colon [\![p]\!](\pi)(h) \overset{\text{morally}}{=} [\![\mathcal{A}_p]\!](\pi{::}h) \; .$$

Correctness formalized in Lean.

Computability & Decidability Results

Let $\mathcal{S} = (S, \oplus, \odot, \mathbf{0}, \mathbf{1})$ be a computable semiring.

Theorem (Quantitative Safety)

Let $s \in S$. If $\oplus$ is idempotent, then

$$\forall \pi : \forall h : [\![p]\!](\pi)(h) \leq s$$

is decidable. If $\mathcal{S}$ is totally ordered, then counterexample packet-history pairs are computable.

Worst-case latencies, failure probabilities, hub-tracking ...

Let $\mathcal{S} = (S, \oplus, \odot, \mathbf{0}, \mathbf{1})$ be a computable semiring.

Theorem (Quantitative Reachability)

Let $\mathbf{s} \in S$. If $\mathbf{1}$ is the greatest element of $\mathbf{s}$, then

$$\exists \pi : \exists h : [\![p]\!](\pi)(h) \geq \mathbf{s}$$

is decidable and witnessing packet-history pairs are computable.

Best-case latencies, bottlenecks, reliability probabilities,...

Case Study

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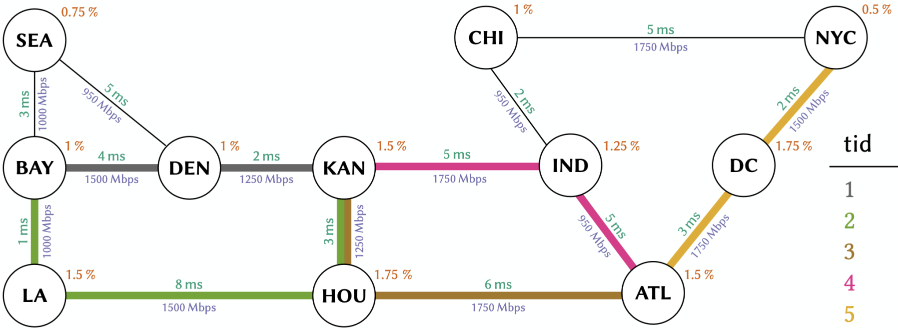

tid	Path
1	BAY, DEN, KAN
2	BAY, LA, HOU, KAN
3	KAN, HOU, ATL
4	KAN, IND, ATL
5	ATL, DC, NYC

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1. A unifying framework for quantitative network verification
2. Semiring-general denotational semantics, computability & decidability results

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Thank you!